Problem 1

(a) Write down the static Maxwell equations in integral form

\[ \oint \vec{D} d\vec{S} = Q = \int \rho_v dV \quad (1) \]
\[ \oint \vec{E} d\vec{l} = 0 \quad (2) \]
\[ \oint \vec{H} d\vec{l} = I = \int \vec{J} d\vec{S} \quad (3) \]
\[ \oint \vec{B} d\vec{S} = 0 \quad (4) \]

(b) Write down the corresponding Maxwell equations in differential form

\[ \nabla \cdot \vec{D} = \rho_v \quad (1) \]
\[ \nabla \times \vec{E} = 0 \quad (2) \]
\[ \nabla \times \vec{H} = \vec{J} \quad (3) \]
\[ \nabla \cdot \vec{B} = 0 \quad (4) \]

(c) Explain in words what each of the equations means

- The number of electric flux lines crossing a closed surface equals the charge enclosed by this surface (1).
- The electrostatic field is conservative: work performed by moving a charge around in an electrostatic field is zero if you return to the starting point (2).
- The line integral of the magnetic field intensity over a closed loop corresponds to the current crossing the area inclosed by this loop (3).
- Magnetic field lines are closed. There are no magnetic monopoles where magnetic field lines start or end (4).

Problem 2

A current \( I \) flows through an infinitely long, infinitely thin cylindrical wire along the z-axis. Use Ampere’s law to find an expression for the magnetic field intensity \( \vec{H} \) surrounding the wire.

\[ \oint \vec{H} d\vec{l} = I \]
since \( \vec{H} = H(\rho) \cdot \vec{a}_\phi \) and \( d\vec{l} = \rho d\phi \cdot \vec{a}_\phi \) we get

\[
\oint H(\rho) \vec{a}_\rho \cdot \rho d\phi \vec{a}_\phi = \oint H(\rho) \rho d\phi = H(\rho) 2\pi \rho = I
\]

\[\Rightarrow H = \frac{I}{2\pi \rho}, \quad \vec{H} = \frac{I}{2\phi \rho} \vec{a}_\phi\]