Problem 1

a) Coulomb’s law in spherical coordinates

\[ \vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0 r^2} \hat{a}_r \]

b) Coulomb’s law in rectangular coordinates

\[ \vec{E}(\vec{r}) = \frac{Q}{4\pi\epsilon_0 (x^2 + y^2 + z^2)^{3/2}} \cdot x\hat{a}_x + y\hat{a}_y + z\hat{a}_z \]

c) \( dV = dx \, dy \, dz \) (rectangular)

d) \( dV = \rho \, d\phi \, d\rho \, dz \) (cylindrical)

e) \( dV = r^2 \sin(\theta) \, d\phi \, d\theta \, dr \) (spherical)

f) \( Q = \int \rho_V \, dV \) or \( \rho_V = \frac{dQ}{dV} \)

g) \( \vec{D} = \epsilon_0 \vec{E} \)

h) Gauss’s law in integral form:

\[ \oint_S \vec{D} \, d\vec{S} = \int_{Vol} \rho_V \, dV \]

Problem 2

a) All four charges contribute to the electric field.

b) The flux through the closed (Gaussian) surface is given by the total charge enclosed by the surface, i.e. only the two charges \( q_1 \) and \( q_2 \) (\( \Psi = q_1 + q_2 \)). The net-flux of the electric field from the charges \( q_3 \) and \( q_4 \) through the Gaussian surface is zero. Therefore the value of the flux through the surface, calculated using only the electric field due to \( q_1 \) and \( q_2 \) is equal to that obtained using the field due to all four charges.

Problem 3

Both charges and the point of interest lies on the y-axis. Only the y-component of \( \vec{E} \) is important here.

\[ E = E_1 + E_2 = \frac{Q}{4\pi\epsilon_0 (y - 1)^2} + \frac{Q}{4\pi\epsilon_0 (y + 1)^2} \]
Since $Q_1 = Q_2 = 4\pi\epsilon_0 \frac{Nn^2}{C}$

$$E = \frac{1}{(y-1)^2} + \frac{1}{(y+1)^2} \frac{N}{C}$$

For $y = 2$ m we get

$$E(y = 2m) = \frac{1}{1^2} + \frac{1}{3^2} \frac{N}{C} = \frac{10}{9} \frac{N}{C}$$

For $y \to \infty$ we get

$$E = \frac{1}{y^2} + \frac{1}{y^2} \frac{N}{C} = \frac{2}{y^2} \frac{N}{C}$$

The field is equivalent to a point charge of $2Q$ at the origin ($y=0$).