Force exerted by a magnetic field

\[ \mathbf{F} = q \left( \mathbf{E} + \mathbf{v} \times \mathbf{B} \right) \]

Lorentz force

Magnetic field does not exert a force on a stationary charge. However, there is a force on a charge in motion with velocity \( \mathbf{v} \).

(magnetic field is a relativistic effect of \( \mathbf{E} \))

+ right hand - left hand

Application: Hall effect:

In a conductor, when \( \mathbf{B} \) is applied perpendicular to \( \mathbf{I} \), a small displacement occurs between the crystalline structure and the conduction electrons.

Although we cannot measure the separation between charges, we can infer it by the presence of a small electric field across the conductor in the direction of \( \mathbf{E} \).

Hall effect is used to measure magnetic fields.
Force on a current carrying wire

Remember: $J = \frac{\rho v}{\xi}$ current density

force since: $dF = da \times B$

and $da = \rho v \cdot dv$

$dF = \rho v \cdot dv \times B = \rho v \cdot x \times B \cdot dv$

$\Rightarrow dF = I \times B \cdot dv$

$= I \frac{dx}{\xi} \times B$

$\int I \cdot dv = I \int d\xi$

$F = \int I \cdot d\xi \times B$

eg. Straight conductor in uniform magnetic field

$\vec{B} = -B_x \hat{a}_x$

$dl = dz \cdot \hat{a}_z$

$\Rightarrow F = \int I \cdot d\xi \cdot \hat{a}_z \times (-B_x \cdot \hat{a}_x)$

$= \int -B_x \cdot dz \cdot \hat{a}_z \cdot \hat{a}_y$

$= -\int B_x \cdot dz \cdot \hat{a}_z \cdot \hat{a}_y = -LI \cdot a_y$
In general

\[ F = I \times L \times B \]

Magnitude of force \( F = BIL \cdot \sin \theta \)

Force between two (inclined) parallel wires:

[Diagram showing two parallel wires with currents in opposite directions]

- Equal but opposite currents: wires repel each other
- Same direction of current flow: wires attract each other

\[ H = \frac{I}{2\pi a} \] around one wire

\[ F = \mu I \cdot H = \frac{\mu_0 I^2}{2\pi a} \]

Direction of \( I \Rightarrow \) direction of motion of positive charge

\( \Rightarrow \) use right hand rule

(for negative charges such as electrons use left hand rule)

Force on a closed circuit

\[ \vec{F} = \oint I \, dl \times \vec{B} = -I \oint B \, dl \times d\ell \]
$B$ is uniform across the closed loop, we can take it out of the integral:

$$\mathbf{F} = -I \mathbf{B} \times \mathbf{d}$$

$$= 0$$

to show that $\mathbf{F}$ is the $\mathbf{0}$.

If the field is not uniform, the total force need not be zero.

$$\int_{\mathcal{C}} \mathbf{B} \cdot d\mathbf{l}$$

torque $I = \mathbf{R} \times \mathbf{F}$ is not zero

total force is zero but loop will align to $\mathbf{B}$ field

(no more rotation at that stage)

Magnetic dipole moment $\mathbf{d}_m = I \mathbf{dS}$

$[\mathbf{m}] = \text{A} \text{m}^2$
The nature of magnetic materials

To understand the origin of magnetism, we must look at materials at a microscopic level.

Atom:

Electron in orbit is analogous to a small current loop. When an external magnetic field is applied, it experiences a torque. The torque in turn tends to align the magnetic field produced by the orbiting electron with the external magnetic field.

Electron spin:
Electron may have a spin magnetic moment of about $\pm 9.10^{-24}$ A m$^2$.
Alignment with or against the external field is possible ($\pm$).

Nuclear spin:
Usually negligible (but is e.g. used in hospitals for nuclear magnetic resonance imaging).
Depending on the relative contributions of orbital and spin moments, all materials can be classified as follows:

<table>
<thead>
<tr>
<th>Classification</th>
<th>Magnetic Moments</th>
<th>B-values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamagnetic</td>
<td>[m_{orb} + m_{spin} = 0]</td>
<td>[B_{sat} &lt; B_{appl}]</td>
</tr>
<tr>
<td>paramagnetic</td>
<td>[m_{orb} + m_{spin} = \text{small}]</td>
<td>[B_{sat} &gt; B_{appl}]</td>
</tr>
<tr>
<td>ferromagnetic</td>
<td>[</td>
<td>m_{spin}</td>
</tr>
<tr>
<td>anti-ferromagnetic</td>
<td>[</td>
<td>m_{spin}</td>
</tr>
<tr>
<td>ferrimagnetic</td>
<td>[</td>
<td>m_{spin}</td>
</tr>
<tr>
<td>super-paramagnetic</td>
<td>[</td>
<td>m_{spin}</td>
</tr>
</tbody>
</table>

**Qualitative Description:**

If the magnetic moments in a material are randomly distributed, there is no overall magnetic moment. As a result of the application of a magnetic field, the magnetic dipoles may be aligned to produce a macroscopic dipole moment.

\[
M = \lim_{\delta V \to 0} \frac{1}{\delta V} \sum_{i=1}^{n} m_i 
\]

(Magnetization)

Compare with polarization due to \[E\]
where \( \mathbf{M} = \chi_m \cdot \mathbf{H} \)

and thus \( \mathbf{B} = \mu_0 (\mathbf{H} + \chi_m \cdot \mathbf{M}) \)

\[ = \mu_0 (\mathbf{H} + \chi_m \cdot \mathbf{H}) \]

\[ = \mu_0 (1 + \chi_m) \mathbf{H} \]

\[ = \mu_0 \cdot \chi_r \cdot \mathbf{H} = \mathbf{B} \cdot \mathbf{H} \]

\( \chi_r = \text{relative permeability} \)
\( \mu = \text{permeability} \)

(tabulated in appendix C, e.g. iron: \( \chi_r = 4000 \))

Ferromagnetic materials

random alignment of domains

In an externally applied \( \mathbf{B} \)-field the magnetic moments line up with \( \mathbf{B} \).

\( \mathbf{B}_{\text{int}} > \mathbf{B}_{\text{applied}} \)
A d orbital electron in an atom has an orbital magnetic dipole moment and a spin magnetic dipole moment.

These two vectors combine vectorially.

The resultant from each atom combines with those for all the other atoms in a sample of material.

If the combination of all these magnetic dipole moments produces a magnetic field, then the material is magnetic.

There are three general types of magnetism:

**Dia magnetism**
- Weak magnetic dipole moments are produced in the atom, when material is placed in an external B-field
- Only a feeble net magnetic field
- Dipole moments disappear when B-field is removed
- All common making exhibit diamagnetism. Only those that do not exhibit the following forms of magnetism are called diamagnetic

**Paramagnetism**
- Atoms where the effects of the electron spin and orbital motion do not quite cancel
- In an external B-field those individual magnetic moments are aligned and cause net magnetic field
- Alignment + field disappears if B-field is removed
Ferromagnetism (what we typically think of magnets)
- Property of iron, nickel, ...
- Regions with strong magnetic dipole moments (domains)
- Bext can align the magnetic moments of these regions \(\Rightarrow\) strong magnetic field
- Partially persists when Bext is removed! (hysteresis)

A diamagnetic material placed in an external magnetic field Bext develops a magnetic dipole moment directed opposite Bext. If the field is non-uniform, the diamagnetic material is repelled from a region of greater magnetic field toward a region of lesser field.

A paramagnetic material placed in an external magnetic field Bext develops a magnetic dipole moment in the direction of Bext. If the field is non-uniform, the paramagnetic material is attracted toward a region of greater magnetic field.

Same is true for ferromagnetic material
(only the magnetic dipole is much stronger)
Ferromagnetic hysteresis

Field due to magnetic moment

$B_m \rightarrow B_0$ (external field)

even though there is no more external field the material remains magnetised