Problem 1 Write down the following relations or laws:

(a) Vector force between two point charges \( q \) and \( Q \) at a distance \( r \)

(b) Definition of the electric field

(c) Gauss’s law

(d) Work performed by moving a charge \( q \) in a field \( \vec{E} \) from point P1 to P2

(e) Electric potential of a point charge \( Q \)

(f) Relation between electric field and potential

(g) Relation between current and vector current density

(h) Continuity equation of electrostatics

(i) Relation between electric field and electric flux density in dielectric

(j) Boundary conditions for \( \vec{E} \) and \( \vec{D} \) between two dielectrics

(k) Definition of capacitance

(l) General form of Ohm’s law
Problem 2

Two point charges $Q_1$ and $Q_2$ are located at $\vec{r}_1 = (1,3,2)$ m and $\vec{r}_2 = (-1,5,3)$ m. Both charges have the same charge $Q_1 = Q_2 = 5 \text{nC}$.

(note: $\epsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m}$)

(a) Calculate the force $\vec{F}_{12}$ that $Q_1$ exerts on $Q_2$

(b) What is the force that $Q_2$ exerts on $Q_1$?

(c) What is the potential of both charges at the origin?

(d) If you place a third charge $Q_3 = Q_1 = Q_2$ at the origin it will be accelerated and moves away from the other two charges and will approach a final velocity. Find the kinetic energy that the charge will have when it is infinitely far away from the origin.

(assume that $Q_1$ and $Q_2$ are fixed in space and the potential $V=0$ infinitely far away from the origin).
Problem 3

Consider an arrangement of two conducting, concentric, infinitely long cylinders with a symmetry axis along the z-axis. The inner conductor is solid and has a radius $a$. The outer conductor is infinitely thin (sheet) and has a radius $b > a$ (as illustrated in the figure). A section of this system of length $L$ has a charge $+Q$ on the inner conductor and $-Q$ on the outer conductor.

(a) Use Gauss’s law to derive an expression for the electric field between the inner and the outer conductor ($a < \rho < b$).

(b) Use Gauss’s law to derive an expression for the field outside the outer cylinder ($\rho > b$).

(c) How does the field between the two conductors ($a < \rho < b$) change if you double the charge on the outer conductor?

(d) What can you say about the electric field inside the solid inner conductor ($\rho < a$)?
Problem 4

(a) State and explain the divergence theorem for the electric flux density $\vec{D}$.

$\nabla \cdot \vec{D}$ in cylindrical coordinates is given by

$$\nabla \cdot \vec{D} = \frac{1}{\rho} \frac{\partial}{\partial \rho} (\rho D_\rho) + \frac{1}{\rho} \frac{\partial D_\phi}{\partial \phi} + \frac{\partial D_z}{\partial z}.$$  

The electric flux at any point is given as

$$\vec{D} = 2\rho \cdot \cos \left(\frac{\phi}{2}\right) \cdot \hat{a}_\rho + 6\rho \cdot \sin \left(\frac{\phi}{2}\right) \cdot \hat{a}_\phi + \rho^2 \cdot \hat{a}_z \quad \text{C/m}^2$$

(b) Find an expression for the charge density $\rho_V$ at any point in space.

(c) Evaluate $\rho_V$ at the point $P (\rho = 1, \phi = \pi, z = \sqrt{2})$.

(d) What is the total charge enclosed by the surface $\rho = 1, \phi = \pi, \phi = 2\pi, z = -1$ and $z = 1$?

(e) What is the total electric flux crossing this surface?
Problem 5

The electric potential at any point in space is given in spherical coordinates as

\[ V = \frac{Q \cdot d}{4\pi\epsilon_0 \cdot r^2 \cos(\theta)} \cdot \frac{V}{m}, \]

where \( d \) is a constant.

The gradient of a scalar field \( A \) in spherical coordinates is given as

\[ \vec{\nabla} A = \frac{\partial A}{\partial r} \hat{a}_r + \frac{1}{r} \frac{\partial A}{\partial \theta} \hat{a}_\theta + \frac{1}{r \cdot \sin(\theta)} \frac{\partial A}{\partial \phi} \hat{a}_\phi \]

(a) Find the vector electric field at any point

(b) How is the field different from that of a point charge at the origin?

(c) Find an expression for the work done in carrying a charge \( q \) from the point \( P1 = (r = 2 \text{ m}, \phi = 10^\circ, \theta = 90^\circ) \) to the point \( P2 = (r = 5 \text{ m}, \phi = 100^\circ, \theta = 90^\circ) \)

(d) Repeat for \( P2 = (r = 2 \text{ m}, \phi = 190^\circ, \theta = 90^\circ) \)?

(e) What will happen to an electron that is placed at \( P1 \) if \( Q \) and \( d \) are both positive?
Problem 6

A plate capacitor (no dielectric between the plates) with a capacitance of 4 pF and an area of 1 square millimeter per plate is connected to a 12 V battery. The capacitance of such a system is calculated as $C = \epsilon S/d$, where $S$ is the area and $d$ the separation between the plates.

(note: $\epsilon_0 = 8.85 \cdot 10^{-12} \text{ F/m}$, and $e = 1.602 \cdot 10^{-19} \text{ C}$)

(a) Calculate the work required to move one electron from the positive plate to the negative plate?

(b) How does the capacity change if you fill the entire space between the two plates with silicon ($\epsilon_r = 11.8$) ?

(c) How does this affect the voltage (=potential) across the capacitor?

(d) You now charge the capacitor with the 12 V battery and without the dielectric, then disconnect the battery. If you insert the same dielectric, how does the voltage across the capacitor change?

(e) What happens if you could now pull the two plates apart to double the distance $d$ and why?