Solutions for EE2 Midterm I

[1].

(a)

(b) A simple cubic lattice has 1 atom per unit cube. Therefore, we have 1 atom for every $a^3$ volume of material OR

$60/6.023 \times 10^{23}$ g for every $(5 \times 10^{-8})^3$ cm$^3$ of material OR

density of metal = $60/6.023 \times 10^{23}/(5 \times 10^{-8})^3 = 0.797$ g cm$^{-3} = 797$ kg m$^{-3}$. 
\[ n_i = \sqrt{N_c \ast N_v} \ast e^{-E_g / 2k_BT} \]
\[ = 2.54 \ast 10^6 \text{ cm}^{-3} \]

(a) Valence band; positive charges.
(b) Yes it will. If the band gap is large, more energy is required for electrons to jump from the valence band to the conduction band. Fewer holes will be created if fewer electrons escape the valence band. This results in a decrease in the number of holes and the density of holes. Also, \( p_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right) \) so \( p_i \) is a function based on the band gap \( E_g \).
(c) \[ p_i = \int_{E}^{E_f} \frac{Z_v(E)f(E)dE}{V} \]
(d) \( p_i = n_i = 3.3997 \times 10^{12} \text{ cm}^{-3} \)
(e) The probability that a state is occupied by a hole is equal to the probability that a state is NOT occupied by an electron
\[ 1 - f(E) = 1 - \frac{1}{1 + \exp \left( \frac{E - E_F}{kT} \right)} \]

\[ = 1 - \frac{1}{1 + \exp \left( \frac{E_F - 2kT - E_F}{kT} \right)} \]

\[ = 1 - \frac{1}{1 + \exp \left( \frac{-2kT}{kT} \right)} \]

\[ = 1 - \frac{1}{1 + e^{-2}} \]

\[ = 0.1192 \]

\[ [4]. \]

\[ n_i = \sqrt{N_C N_V} \exp \left[ - \frac{E_g}{2kT} \right] \]

At 300K,

\[ \sqrt{N_C N_V} = \frac{n_i}{\exp \left[ - \frac{E_g}{2kT} \right]} = 10^{10} \exp \left[ \frac{1.12}{0.0259} \right] = 2.455 \times 10^{19} \text{cm}^{-3} \]

Assuming \( N_C \) \& \( N_V \) are constant with \( T \), we have \( n_i = 10^{13} \text{cm}^{-3} \) at a \( T \) given by:

\[ \frac{10^{13}}{10^{10}} = \exp \left( - \frac{1.12 \times 1.602 \times 10^{-19}}{2 \times 1.38 \times 10^{-23}} \left( \frac{1}{T} - \frac{1}{300} \right) \right) \]

\[ \Rightarrow T_2 = 440.4K \]

Now using the exact formula, \( n_i (440.4K) \) is given by:

\[ n_i = 2.455 \times 10^{19} \times \left( \frac{440.4}{300} \right)^{3/2} \exp \left[ - \frac{1.12 \times 1.602 \times 10^{-19}}{2 \times 1.38 \times 10^{-23} \times 440.4} \right] = 1.7 \times 10^{13} \text{cm}^{-3} \]

Since the solution is remarkably close to 440.4K, we try to compute \( n_i \) for a few temperatures around 440.4K as below:

At \( T = 435K \), \( n_i = 1.39 \times 10^{13} \text{cm}^{-3} \).

At \( T = 430K \), \( n_i = 1.14 \times 10^{13} \text{cm}^{-3} \).
At $T = 425\text{K}$, $n_i = 7.7 \times 10^{12} \text{ cm}^{-3}$.

At $T = 427\text{K}$, $n_i = 1.02 \times 10^{13} \text{ cm}^{-3}$. (which is a close enough answer).

Any answer in the range $422 \text{K} < T < 432 \text{K}$ is an acceptable answer.

Thus, this semiconductor device will cease to operate if $T > 427\text{K}$ or 154 °C.