Consider an extrinsic compensated Si sample with \( N_D = 10^{17} \text{ cm}^{-3} \) of phosphorus atoms and \( N_A = 2 \times 10^{17} \text{ cm}^{-3} \) of boron atoms. Assume that the ionization energies for phosphorus and boron are 44 meV and 45 meV respectively and that \( E_I \) is at the mid-gap. Give all energy level locations relative to \( E_I \). (\( n_i = 1.45 \times 10^{10} \text{ cm}^{-3} \))

(a) At \( T = 0K \), what is the location of \( E_F \)?

(b) At \( T = 300K \), what is the location of \( E_F \)?

(c) At \( T = 500K \), what is the location of \( E_F \)?

(d) Sketch \( E_F \) vs temperature from 0K to very high temperatures. Indicate (on your plot) the freeze-out, extrinsic, exhaustion and intrinsic regions.

Solution:

(a) Since both acceptors and donors are present at the same time and \( N_A > N_D \), compensation occurs and we are finally left with \( N_A - N_D = 10^{17} \text{ cm}^{-3} \) uncompensated acceptors.

Compensation happens even at \( T = 0K \) and the topmost filled (whether completely or partially) band is the acceptor level. Hence the Fermi level, \( E_F = E_V + 0.045 \text{eV} \). Since \( E_i = 0.56 \text{eV} \) above \( E_V \), we have \( E_F = E_I - 0.515 \text{eV} \)

(b) At \( T = 300K \), \( E_F = E_I - kT \ln \left( \frac{p_{p0}}{n_i} \right) \)

Therefore, \( E_F = E_I - 0.02584 \times \ln \left( \frac{10^{17}}{1.45 \times 10^{10}} \right) = E_I - 0.4069 \text{eV} \).

(c) At \( T = 500K \),

\( n_i (500K) = n_i (300K) \times (500/300)^{3/2} \times \exp \left[ -1.12 \times 1.602 \times 10^{-19} / 2 / 1.38 \times 10^{-23} \left( \frac{1}{500} - \frac{1}{300} \right) \right] = 1.814 \times 10^{14} \text{ cm}^{-3} \).

Therefore, this semiconductor is still p-type with \( p_{p0} = 10^{17} \text{ cm}^{-3} \) and \( E_F = E_I - kT \ln \left[ \frac{p_{p0}}{n_i} \right] = E_I - 0.2719 \text{eV} \).

(d) See diagram below:
Consider a sample of semiconductor as shown in the diagram below:

a) How long does it take for electrons (on average) to drift 1 µm in pure Si at an electric field of 100V/cm given that \( \mu_n = 1350 \text{ cm}^2/\text{Vs} \)?

b) If we dope the sample with \( 10^{17} \text{ cm}^{-3} \) Phosphorus atoms, then what is the new mobility in (cm\(^2\)/V sec)? What is the scattering time for this doped Si?

c) What is the diffusion constant in this situation?

d) If the (same doped) sample is 0.1 cm long, has a 100 µm\(^2\) cross sectional area and is subject to 10 V applied across the length of the bar at room temperature, what is the value of the electric field in the bar?

e) Is there any current in the sample? If so, what type of current is it?
f) What is the value of current? What is the current density in the sample?

Solution:
(a) \[ \left| \nu_d \right| = -\mu_n \times E \] then \( \nu_d = 1350 \times 100 = 1.35 \times 10^5 \text{ cm/sec} \) then \( t = 1 \times 10^{-4} \text{ cm}/1.35 \times 10^5 = 0.74 \text{ ns} \)

(b) From the table and graph the mobility \( \mu_n = 750 \text{ cm}^2/\text{V sec} \) then we will get: \( \tau_{cn} = m \times \mu_n / q = 9.11 \times 10^{-31} \times 750 / 1.6 \times 10^{-19} = 4.27 \text{ ns} \)

(c) By Einstein’s equation: \( D_n = \mu_n \times kT / q = 19.425 \text{ cm}^2/\text{sec} \)

(d) \( V = -E \times d \) then \( |E| = V/d = 10/0.1 = 100 \text{ V/cm} \)

(e) Yes only drift current.

(f) Conductivity \( \sigma = q \times \mu_n \times n_0 = 1.6 \times 10^{-19} \times 750 \times 10^{17} = 12 / \Omega \text{ cm} = 1/\rho = 1/\text{resistivity} \)

Therefore \( \rho = 0.833 \Omega \text{ cm} \) so resistance \( R = \rho \times L/A = 0.833 \times 0.1/10^{-6} = 8330 \Omega \)

And \( I = V/R = 10/8330 = 1.20 \text{ mA} \)

\( J_n = q \times \mu_n \times n \times E = 1200 \text{ A/cm}^2 \)

or \( J_n = I/A = 1.2 \text{ mA} / 10^{-6} = 1200 \text{ A/cm}^2 \) which is the same as the above.

[3].
Consider the following energy band diagram. Assume that the semiconductor to be Si at 300K with \( E_i - E_F = E_g/4 \) at \( x = \pm L \) and \( E_F - E_i = E_g/4 \) at \( x = 0 \).

\[ \text{Graphical representation of energy band diagram} \]

a. Sketch the electrostatic potential (V) inside the semiconductor as a function of x. State your reasons.

b. Sketch the electric field \( E \) inside the semiconductor as a function of x. State your reasons.

c. Determine the majority carrier concentration of the \( x > L \) portion of the semiconductor. Show all your steps.

d. Give a quantitative sketch of \( n \) and \( p \) versus \( x \) inside the sample. State your reasons.

e. Determine the resistivity of the \( x > L \) portion of the semiconductor. Show all your steps.

Solution:
(a) Positive voltages create negative potential energies and cause the energy bands to bend downwards.
If the arbitrary voltage $g$ reference point is taken to be $V = 0$ at $x = L$, then

\[
\varepsilon = \frac{1}{q} \frac{dE}{dx}
\]

(b) Since \( \exp \left( \frac{F_i}{E_n kT} \right) \) and \( \exp \left( \frac{E_i - E_F}{kT} \right) \), we have

\[
n = n_i \exp \left( \frac{E_F - E_i}{kT} \right) \quad \text{and} \quad p = n_i \exp \left( \frac{E_i - E_F}{kT} \right)
\]
Based on the EF, we know this is a p-type semiconductor. In the $x > L$ region, $E_i - \frac{E_G}{4} = 0.28$ eV and

$$N_A = p = n_i \exp \left( \frac{E_i - E_F}{kT} \right) = 10^{10} e^{0.28} = 4.96 \times 10^{14} / \text{cm}^3$$

From the table, $\mu_p = 480 \text{ cm}^2/\text{Vs}$, and therefore,

$$\rho = \frac{1}{q \mu_p N_A} = \frac{1}{(1.6 \times 10^{-19})(480)(4.96 \times 10^{14})} = 26.3 \text{ ohm-cm}$$

Consider a sample of Si that is uniformly doped with $10^{16}$ cm$^{-3}$ donors. Assume that this sample was being uniformly illuminated by optical radiation for the last 3 hours with an excess optical generation rate of $10^{21}$ cm$^{-3}$s$^{-1}$. There are $10^{15}$ cm$^{-3}$ bulk recombination centers at $E_i$ with electron and hole capture cross-sections of $10^{-14}$ cm$^2$. The thermal velocity of both electrons and holes is $10^7$ cm$^{-1}$s$^{-1}$.

At $t = 0$ (defined as present moment), the light is turned OFF and it is known that there is no current flowing through the device.

a. Calculate the average life time, $\tau_p$, of holes.
b. Calculate the electron and hole concentrations immediately before $t = 0$.
c. Compute the total hole concentration at $t = 5\tau_p$ and $25\tau_p$.
d. What is the total electron concentration at $t = 5\tau_p$ and $25\tau_p$?

**Solution:**

First, we note that $n_{0} = 10^{16}$ cm$^{-3}$ and $p_{0} = 2.1025 \times 10^4$ cm$^{-3}$.

(a) Then, we compute $\tau_p \approx \tau_{p0} = 1/(N_p \sigma_p \nu_{th}) = 10$ ns.
(b) Before $t = 0$, light has been turned on for a long time (3 hours $>> 10$ns), we have
\[ \delta p_{ss} = G_{opt} \tau_p = 10^{13} \text{ cm}^{-3} = \delta n_{ss}. \]
Therefore, $p = p_{n0} + \delta p_{ss} \approx 10^{13} \text{ cm}^{-3}$ and $n = n_{n0} + \delta n_{ss} \approx 10^{16} \text{ cm}^{-3}$.
(c) After $t = 0$ when the light is turned off, the excess carriers decay exponentially with a time constant $\tau_p = 10$ns.
Therefore, $\delta p(t) = \delta p_{ss} \exp(-t/\tau_p)$.
So, $\delta p(5\tau_p) = 10^{13} \exp(-5) = 6.74 \times 10^{10} \text{ cm}^{-3}$ and $\delta p(25\tau_p) = 10^{13} \exp(-25) = 1.39 \times 10^{2} \text{ cm}^{-3}$.
Therefore, $p(5\tau_p) = p_{n0} + \delta p(5\tau_p) \approx 6.74 \times 10^{10} \text{ cm}^{-3}$ and $p(25\tau_p) = p_{n0} + \delta p(25\tau_p) \approx 2.1025 \times 10^{4} \text{ cm}^{-3}$.
(d) We note that $\delta p(t) = \delta n(t)$.
Therefore, $n(5\tau_p) = n_{n0} + \delta n(5\tau_p) \approx 10^{16} \text{ cm}^{-3}$ and $n(25\tau_p) = n_{n0} + \delta n(25\tau_p) \approx 10^{16} \text{ cm}^{-3}$.