Diffusion

Carrier can move via diffusion as well. As in all cases, there is a driving force to move particles from a high concentration region towards a low concentration region.

In general we have the "Fick's law"

\[ \text{Flux} = -D \nabla n \]

where \( D \) is called the diffusion constant.

For electrons, current density due to diffusion is given by

\[ J_n = (-g) - D_n \nabla n \]

\[ = g D_n \nabla n \]

On \( \nu = 1-D \) \( J_n = 8 D_n \frac{\partial n}{\partial x} \)
For hole, the current due to diffusion is:

\[ J_p = (+q) \cdot -D_p \nabla p \]

\[ = (-q) \cdot D_p \nabla p \]

\[ \text{note the sign.} \]

\[ J_p = -qD_p \frac{\partial p}{\partial x} \]
When both drift and diffusion exist (i.e. when there is an E-field as well as concentration gradient), the two components sum:

\[ J_n = g n \mu_n \mathbf{E} + g D_n \frac{\partial n}{\partial x} \]

\[ J_p = g p \mu_p \mathbf{E} - g D_p \frac{\partial p}{\partial x} \]

(Note the signs)

And the total \( J \) is \( J = J_n + J_p \).

In fact, it can be derived from thermal dynamics that:

\[ J_n = -g n \mu_n \frac{\mathbf{E} \cdot \mathbf{E}}{2x} \]
(Recall \( E_{\text{Eu}} \) is simply the electrochemical potential of electrons)

But for non-degenerate case

\[
N = N_c e^{-\frac{(E_o - E_F)}{kT}}
\]

\[
\Rightarrow E_o - E_F = kT \ln \frac{N_c}{n}
\]

\[
\Rightarrow E_F = E_o - kT \ln \frac{N_c}{n}
\]

\[
\therefore \frac{2E_{\text{Eu}}}{g} = \frac{1}{\beta} \frac{2E_o}{2k} + \frac{kT}{g} \frac{2\beta n N_c}{n}
\]

Recall \( \varepsilon = \frac{1}{\beta} \frac{2E_o}{2k} \)

And \( \frac{kT}{g} \frac{2 \ln(n N_c)}{2k} = -\frac{kT}{g} n \frac{2n}{n} \frac{2n}{n} \)
Together we have

\[ J_n = -\xi n \mu_n \left( -3 - \frac{\hbar n}{8} \frac{2n}{3x} \right) \]

\[ = \xi n \mu_n (\Xi + \Phi(x)) \mu_n \frac{2n}{3x} \]

Compared to our expression earlier:

\[ J_n = \xi n \mu_n \Xi + \Phi D_n \frac{2n}{3x} \]

We set \( D_n = \frac{\hbar x}{\mu n} \) (Einstein relation)

for non-degenerate semicord.

Similarly, \( \frac{D_p}{\mu p} = \frac{\hbar x}{\xi} \).
Semiconductor carrier statistics in non-equilibrium.

Consider the case that light is shining on a piece of semiconductor. We have the following reaction:

\[ \text{light} \rightarrow \text{exciton} \]

i.e. light excites an electron in the valence band into the conduction band, creating an electron-hole pair (often called electron-hole pair).
If the rate of this happening is $G_n$

\[
\frac{dn}{dt}\bigg|_{\text{gen}} = \frac{dp}{dt}\bigg|_{\text{gen}} = G_n
\]

\[P \downarrow: \text{NP will increase due to } G_n\]

If nothing counteracts this, as $t \to \infty$, $n \to \infty$, $p \to \infty$. Of course this is NOT physical!

There is also a process by which an electron in the conduction band "jumps" to an empty state in the valence band, thus eliminating one and a hole
\[ e + h \rightarrow \Phi \]

The rate of this process (c.f. Chem 1) is proportional to \( n \cdot p \) (i.e. \( \Sigma e \Sigma h \Sigma \))

\[ R = rnp \]

In fact, under thermal equilibrium

\[ G_{\text{he}} = R_{\text{he}} = Rn_0p_0 = Rn^2 \]

Now, how does recombination occur?

(I) Direct recombination

\[ \text{Diagram:} \]

\[ \text{Diagram:} \]
can happen for direct bandgap semiconductors (i.e. conduction band minimum = valence band maximum are both at \( \mathbf{k} = 0 \)) e.g. GaAs, InP, ...

In this case

\[
R = \tau (n_0 + \Delta n)(p_0 + \Delta p)
\]

\[
= \tau n_0 p_0 + \tau (n_0 \Delta p + p_0 \Delta n) + \tau n_0 \Delta p
\]

Under low-level injection (i.e.

\( \Delta n, \Delta p \ll \) the equilibrium majority carriers.

e.g. \( n \)-type. \((n_0 \gg p_0)\)

\[
R \cong \tau n_0 p_0 + \tau n_0 \Delta p
\]

\[
U = R - G_{th} = \tau n_0 \Delta p = \frac{\Delta p}{\tau}
\]

i.e. \( \tau \), (which is called the lifetime) = \( \frac{1}{n_0} \)
meaning of $\varepsilon$:

Say at $t = 0$ $\Delta p = \Delta p^*$ and the force to generate $\Delta p$ is removed.

$$\frac{\Delta p}{2\varepsilon} = -U = \frac{\Delta p}{\varepsilon}$$

$$\Rightarrow \Delta p = \Delta p^* e^{-\frac{t}{\varepsilon}}$$

What about non-direct bandgap semiconductors (conduction band minimum, $E_c$, and valence band maximum $E_V$ at not the same $k$'s)?

e.g. Si, Ge, ...
Direct recombination is not possible to conserve both energy and crystal momentum.

Recombination is via traps, and the theory that describes the recombination is called the SRH theory.

\[ \text{trap} \quad a \quad b \quad c \quad d \]

\[ a: e^- \text{ capture} \]
\[ b: e^- \text{ emission} \]
\[ c: h \text{ capture} \]
\[ d: \text{hole emission} \]
With SRH theory, the recombination is given by

\[ U = \frac{pn - N_i^2}{\tau_p n + \tau_n p} \]

With \( \tau_n, \tau_p \propto \frac{1}{N_t} \)

\( N_t \) is the density of traps.