EE232A – Homework 3

Problem 1. This problem works you through the generation of sample paths of a Poisson process.

a) Consider a transformation of random variables, $Y = -\ln(X)/\lambda$. Find the output PDF when $X$ is a random variable uniformly distributed on $[0, 1]$.

b) Use the result in a) and the `rand` function in Matlab to generate a Poisson arrival process, $N(t)$.

c) Produce and plot a sample path of $N(t)$ over the interval $(0, 1]$ when $\lambda = 5$.

d) Produce and plot a sample path of $N(t)$ over the interval $(0, 1]$ when $\lambda = 50$.

e) In the two sample paths plotted above what is the probability of the $N(1)$ that you observed?

Note all students might not have the same answer on this problem.

Problem 2. Let $N(t)$ be the counting process of a Poisson process with rate $\lambda$.

a) Find the joint probability mass function of $N(t), N(t+s)$ for $s > 0$.

b) Find $E[N(t)N(t+s)]$.

c) Find the probability that two arrivals come in an arbitrary time interval of length $s$.

d) Find the interval $[t, s]$ which has the greatest probability of having the third and fourth arrival only.

e) If $N(t) = 4$ find $f_{S(4)}(s|N(t) = 4)$ where $S(4)$ is the fourth arrival time. Make a plot comparing $f_{S(4)}(s|N(t) = 4)$ and $f_{S(4)}(s)$ for $t = 1$ and $\lambda = 1$ and $\lambda = 4$.

Problem 3. Let $N_1(t)$ and $N_2(t)$ be two independent Poisson processes with rate $\lambda_i$ and let $N(t) = N_1(t) + N_2(t)$.

a) If $N(t) = 1$ then show that $P(N_1(t) = 1 | N(t) = 1) = \frac{\lambda_1}{\lambda_1 + \lambda_2}$ and $P(N_2(t) = 1 | N(t) = 1) = \frac{\lambda_2}{\lambda_1 + \lambda_2}$. Note these are the probabilities that the first arrival comes from $N_i(t)$ for $i = 1, 2$.

b) If $N(t) = n$ and $N_1(t-\tau) = k-1$ and $N_2(t-\tau) = n-k$ for $\tau > 0$ then show that $P(N_1(t) = k | N(t) = n, N_1(t-\tau) = k-1, N_2(t-\tau) = n-k) = \frac{\lambda_1}{\lambda_1 + \lambda_2}$. Note this is the probability that the $n^{th}$ arrival comes from $N_1(t)$.

c) If $N(t) = n$ find $P(N_1(t) = k | N(t) = n, )$, i.e., the probability that exactly $k$ of the $n$ arrivals came from $N_1(t)$.

d) What is the probability that $n$ arrivals occur in $N_1(t)$ before $n$ arrivals occur in $N_2(t)$?

Problem 4. Consider a Poisson random variable, $N$, with parameter $\gamma_N$.

a) Find the characteristic function of $N$.

b) Find $E[N^2]$ and var ($N$).

c) If $Q$ is another Poisson random variable with parameter $\gamma_Q$ show that $Y = N + Q$ is also Poisson and identify $\gamma_Y$. 

Problem 5. Gallagher 2.6

Problem 6. Gallagher 2.18 – In honor of the electoral process we all get to experience!

Problem 7. Gallagher 2.19