1. Consider a binary detection problem with 
\[ p_0(x) = g(x; 0), \quad p_1(x) = \frac{1}{2} g(x; -1) + \frac{1}{2} g(x; 1), \quad x \in \mathbb{R}, \]
where 
\[ g(x; \mu) = (2\pi)^{-1/2} e^{-(x-\mu)^2/2}. \]
Assume equiprobable hypotheses, and \( C_{00} = 0, \ C_{11} = 0, \ C_{10} = 1, \ C_{01} = 1. \)

(a) [10 points] Compute the likelihood ratio function, \( \Lambda(x). \)

(b) [10 points] Write the expressions for the decision regions, \( \mathcal{R}_0 \) and \( \mathcal{R}_1. \) Go as far as you can.

(c) [10 points] Under the Neyman-Pearson criterion, where \( P_{FA} = 2 \times 10^{-2}, \) what is the achievable \( P_D? \)

2. Consider a binary detection problem with 
\[ p_0(x) = K_0 e^{-x}, \quad p_1(x) = \frac{K_1}{\sqrt{2\pi}} e^{-(x-4)^2/2}, \quad 0 < x < \infty, \]
\[ p_0(x) = p_1(x) = 0, \quad x < 0. \]

(a) [5 points] Determine the values of the constants \( K_0 \) and \( K_1. \)

(b) [10 points] Compute the likelihood function \( \Lambda(x). \)

(c) [10 points] Assume Maximum Likelihood detection and \( Q(4) \approx 0. \) The value \( x = 1.9 \) is received. What is the output of the detector (i.e., is it \( \mathcal{H}_0 \) or \( \mathcal{H}_1)? \)

3. Consider a binary detection problem with \( p_0(x) \) and \( p_1(x) \) as in Fig. 1(a) and (b) respectively.

(a) [5 points] Compute the likelihood ratio function \( \Lambda(x). \)

(b) [10 points] With the choice of Bayesian costs \( C_{00} = C_{11} = 0, \ C_{10} = C_{01} = 1, \) compute the minimax cost, \( P_{FA} \) and \( P_M. \)

(c) [10 points] Compute minimax cost, \( P_{FA} \) and \( P_M \) when the costs are \( C_{00} = 3, \ C_{11} = 0, \ C_{10} = 4, \ C_{01} = 1. \)
Figure 1: (a): $p_0(x)$, (b): $p_1(x)$