1. The closed-loop poles are obtained by solving the characteristic equation:
\[ P_{ch}(s) = 1 + G_{td}G_p = 0 \Rightarrow s^4 + 6s^3 + 11s^2 + 6s + K = 0. \]
We draw up the RH table:

| s^4 | 1 | 11 | K |
| s^3 | 6 | 1 |   |
| s^2 | 10 | 1 | - |
| s^1 | 10 - K | K | - |
| s^0 | K | - | - |

In order for the closed-loop to not have any unstable poles, we need to ensure that there are no sign-changes in the first column of the RH table. Hence, \( K < 10 \) or \( K > 0 \); i.e., \( K \in (0, 10) \).

2. (i) You should check and mark on the root locus the following: 2 branches; 1 asymptote: along -ve real axis; break-away point: \( s = -0.5858 \); break-in point \( s = -3.414 \); no cross-over point. Figure attached:

![Root Locus](image.png)

Figure 1: Root Locus for 2[i]
(ii) You should check and mark on the root locus the following: 3 branches; 3 asymptotes: \(\{60^\circ, 180^\circ, 300^\circ\}\) through \(G = -5.33\); break-away point: \(s = -2.92\); cross-over point: \(s = \pm j8.24\). Figure attached:

![Root Locus Figure](image)

Figure 2: Root Locus for 2(ii)

Note: to figure out cross-over points, we construct the RH table from the characteristic equation:

\[
1 + \frac{K}{(s+2)(s+4)(s+10)} = 0 \Rightarrow P_{ch}(s) = s^3 + 16s^2 + 68s + (80 + K) = 0.
\]

\[
\begin{array}{c|ccc}
   s^3 & 1 & 68 \\
   s^2 & 16 & 80+K \\
   s^1 & 16-80-k & - \\
   s^0 & 80 + K & - \\
\end{array}
\]

We know that if the RoLo has a pair of cross-over points, then those points are also the purely imaginary poles of the closed-loop. But for that to happen, one of the rows of the RH table must go to zero. Applying the reverse argument, we check for a row that we can set completely to zero. In this case, the \(s^1\) row may be set to zero by setting 80 + \(k\) = 16 \cdot 68. Then the auxiliary polynomial is \(P_{aux}(s) = 16s^2 + (80 + k) = 16s^2 + 16 \cdot 68\). We know that \(P_{aux}\) is a factor of \(P_{ch}(s)\) and hence the roots of \(P_{aux}(s)\) are also roots of \(P_{ch}(s)\) and hence are closed-loop poles. We see that the roots of \(P_{aux}(s)\) are \(\pm j8.24\). These, being purely imaginary, must also be the cross-over points for the root-locus.
3. (i) You should check and mark on the root locus the following: 2 branches; 2 asymptotes: \{90°, 270°\} through \(G = -1.5\); break-away point: \(s = -1.5\); no cross-over point. Figure attached:

![Root Locus Diagram](image)

Figure 3: Root Locus for 3

(ii) All points on the root-locus shown are valid closed-loop poles. In addition, the damping factor associated with each RoLo point, \(S\), is given by \(\cos \theta\) where \(\theta\) is the angle made by the vector \(OS\) with the negative real-axis where \(O\) is the origin. Hence, to find the closed-loop poles with damping factor of \(\frac{1}{\sqrt{2}}\), we sketch the design-region as the two rays at angles of ±45° and note where they intersect the RoLo (note: \(\cos 45° = \frac{1}{\sqrt{2}}\)). A little geometry confirms that the two points are indeed \(s_{1,2} = -0.5 \pm j0.5\). This is also seen in the MATLAB plot.

Note also that with this value of \(\zeta\), the overshoot is 4.31% which is within the 5% limit specified.

4. Closed-loop poles are given by \(1 + \frac{k}{s^2 + (k+2)s + 4k} = 0\) ⇒ \(s^2 + (k+2)s + 4k = 0\). The two poles as a function of ‘\(k\)’ are: \(s_{1,2} = \frac{-(k+2) \pm \sqrt{(k+2)^2 - 16k}}{2}\).

CASE I: \((k + 2)^2 - 16k < 0\). Then, the poles are complex and are given by:

\[ s_{1,2} = \frac{-(k+2) \pm j\sqrt{16k-(k+2)^2}}{2} = x \pm jy \text{ (say)}. \]
Then $2x = -(k + 2)$ and $2y = \pm \sqrt{16k - (k + 2)^2}$

Eliminating $k$ we get, $(x + 4)^2 + y^2 = 8$ which is a circle with center $(-4, 0)$ and radius $2\sqrt{2}$.

At this point, it is trivial to calculate the break out/in points as $s = -4 \pm 2\sqrt{2} = \{-1.17, -6.82\}$.

CASE II: $(k + 2)^2 - 16k > 0$. Then, the poles are real and must lie along the real axis. The problem would become really simple if we could identify what $L(s)$ is. We observe that the characteristic equation can also be re-formatted as $1 + k \frac{s + 4}{s + 2} = 0$. We thus identify $L(s) = \frac{z + 4}{s + 2}$. Plotting the (finite) poles and zeros of $L(s)$, it is immediate that the RoLo will lie inside the segments $(-2, 0)$ and $(-\infty, -4)$. We don’t need to find the break-away, break-in points anymore since we know that the two branches will break out/in where the circle (found above) intersects the real line. Putting all this information together, the exact root locus can then be drawn as the following:

Figure 4: Exact Root Locus for 4

This root locus is similar to the one drawn for 2(i) but this time it is exact in the sense that we have an analytical expression for the root locus. In fact, you can show that anytime $L(s) = \frac{s + z}{s + 2}$ with $z > p, p, z > 0$, the complex part of the root-locus is a circle with center $(-z, 0)$ and radius $\sqrt{z(z - p)}$. 

4