1. Consider the set of signals

\[ s_i(t) = \begin{cases} 
\sqrt{\frac{2E}{T}} \cos(2\pi f_c t + i \frac{\pi}{4}); & 0 \leq t \leq T \\
0; & \text{elsewhere}
\end{cases} \]

where \( i = 1, 2, 3, 4 \), and \( f_c = n_c / T \) for some fixed integer \( n_c \).

(a) What is the dimensionality, \( N \), of the space spanned by this set of signals?
(b) Find a set of orthonormal basis functions to represent this set of signals.
(c) Using the expansion,

\[ s_i(t) = \sum_{j=1}^{N} s_{ij} \varphi_j(t) \quad i = 1, 2, 3, 4 \]

Find the coefficients \( s_{ij} \).
(d) Plot the locations of \( s_i(t), i = 1, 2, 3, 4 \), in the signal space, using the results of parts (b) and (c).

2. Problem 7.19 from Proakis and Salehi.

3. Consider a set of signals:

\[ s_i(t) = \begin{cases} 
\sqrt{\frac{2E}{T}} \cos(2\pi f_c t + i \frac{\pi}{4}); & 0 \leq t \leq T \\
0; & \text{elsewhere}
\end{cases} \]

where \( i = 1, 3, 5, 7 \).

Calculate the union bound on symbol error probability, \( P_e \).

4. Design a likelihood ratio test to choose between:

- \( H_1: p_1(y) = \frac{1}{(2\pi)^{1/2}\sigma} \exp\left[-y^2/(2\sigma^2)\right]; -\infty < y < \infty \)
- \( H_0: p_0(y) = 1/2 ; -1 \leq y \leq 1 \)

\( H_1 \) and \( H_0 \) are the two hypotheses. Using MAP criterion, find out the decision regions in terms of \( y \) and as a function of \( \sigma^2 \). Assume priori probabilities for \( H_0 \) and \( H_1 \) to be equal.