Work all 12 problems.

Problem 1. Determine for each of the following whether or not the discrete-time system is 1) static, 2) linear, 3) time-invariant, 4) causal, 5) BIBO stable.

a. \( y(n) = \cos[x(n) + 1] \).

b. \( y(n) = x(n) \).

c. \( y(n) = |x(n)| \).

d. \( y(n) = (n - 1)x(n - 1) + n \).

Problem 2. Suppose we have an LTI system with

\[
x(n) = (1/4)^nu(n), \quad h(n) = (1/2)^nu(n).
\]

a. Compute the convolution \( y(n) = x(n) * h(n) \), i.e., find an expression for the output.

b. Find \( y(n) \) for this system if the input is changed to

\[
x(n) = (1/5)^nu(n) - (1/4)^nu(n - 5).
\]

Problem 3. Determine the impulse response and unit step response of the systems described by the given difference equation. Assume zero initial conditions.

a. \( y(n) - \frac{1}{2}y(n - 2) = x(n - 1) \).

b. \( y(n) = \frac{1}{2}y(n - 1) - \frac{1}{16}y(n - 2) + x(n) \).
Problem 4. Consider the system described by the following difference equation:

\[ y(n) - \frac{7}{10} y(n - 1) + \frac{1}{10} y(n - 2) = x(n), \]

where,

\[ x(n) = (1/3)^n u(n), \quad y(-1) = 1, \quad y(-2) = 0. \]

a. Write down the characteristic equation for this difference equation and find its roots.

b. Find the homogeneous solution to this system (do not solve for the constants in this part).

c. Find the particular solution to this system.

d. Find the complete (or total) solution to this system.

Problem 5. Compute the z-transform of the following sequences. Remember to specify the region of convergence (ROC).

a. \( x(n) = \delta(n - 1). \)

b. \( x(n) = \alpha^n u(n) + \beta^n u(-n - 1), \quad |0 < \alpha| < 1, \quad |0 < \beta| < 1. \)

c. \( x(n) = (n + 1)(-1)^n u(n). \)

Problem 6. Compute the z-transform of the following signals. Remember to specify the ROC.

a. \( x_1(n) = \left(\frac{1}{4}\right)^n u(n). \)

b. \( x_2(n) = x_1(n - 3). \)

b. \( x(n) = \left(\frac{1}{2}\right)^n u(n) + \left(\frac{1}{2}\right)^{-n} u(-n). \)

Problem 7. Suppose we receive the quantized digital signal

\[ r_q(n) = \text{Round} [A \cos(0.25\pi n + \theta)] \]

where ‘Round’ means the samples are rounded to the nearest integer. The amplitude \( A \) is a constant but we do not know its value. Furthermore, we
do not know that the phase is $\theta = \pi/4$. We can follow the steps below to estimate the value of $A$. [For purposes of calculation let $A$ actually have the value 5.]

**S1.** Multiply $r_q(n)$ by $x_1(n)$ and $x_2(n)$, where $x_1(n) = \cos(0.25\pi n)$ and $x_2(n) = \sin(0.25\pi n)$. Call the results $y_1(n)$ and $y_2(n)$, respectively.

**S2.** Simply add up the values of $y_1(n)$ and $y_2(n)$ for $n = 0, 1, 2, \ldots N - 1$ (some $N$) and take the average of each (divide by $N$) and then multiply the averages by 2. Call the results $z_1$ and $z_2$, respectively.

**S3.** Compute $\sqrt{z_1^2 + z_2^2}$. This is the estimate of $A$.

a. Follow the 3 steps above and estimate $A$ using $N = 4$.

b. Repeat (a) for $N = 8$.

c. From Homework 1 we know that making $N$ multiples of 4 estimates $A$ exactly if the input samples were not quantized. Based on your answers to parts (a) and (b) what would your estimate for $A$ be if $N$ is 4000. Explain why the estimate is not becoming exact even for very large $N$.

**Problem 8.** Sayed 3.8.

**Problem 9.** Sayed 4.9.

**Problem 10.** Sayed 5.1.

**Problem 11.** Sayed 7.1.

**Problem 12.** Sayed 8.2.