Problem 1. Consider the following analog sinusoidal signal: 

\[ x_a(t) = 2 \cos(200\pi t + \pi/4) \]

a. Sketch the signal for \(0 \leq t \leq 40\text{ ms}\).

b. Suppose \(x_a(t)\) is sampled at \(F_s = 600\) samples/sec. Determine the frequency of the discrete-time signal \(x(n) = x_a(nT)\), \(T = 1/F_s\) and show that it is periodic. Specify the period of \(x(n)\) in milliseconds.

Problem 2. Determine the fundamental period (if it exists) for each of the following signals. If the signal is not periodic then write “not periodic.”

a. \(x_a(t) = \cos(2t)\).

b. \(x(n) = 1 + 2\cos(2\pi n + \pi/6)\).

c. \(x(n) = \sin(2n - \pi)\).

d. \(x(n) = 3\cos(\pi n) + \sin(2\pi n/5 + \pi/6) - \cos^2(\pi n/4 + \pi)\).

Problem 3. Suppose the analog signal 

\[ x_a(t) = 10 \sin(2\pi t + \pi/4), \quad 0 \leq t \leq 2 \]

is input to a 3-bit A/D converter (so it has 8 levels) and two’s complement arithmetic is used. The sampling rate is one sample every \(1/5\) seconds and the level spacing is \(10/3\). You may assume that the quantizer in the A/D works by rounding the sampled value to the nearest level.

a. Draw the waveform \(x_a(t)\) on a graph and indicate the sampled values.

b. Construct a table showing the quantized level of each sampled value and give the two’s complement representation of each of the quantized levels.

c. On a separate graph use the quantized samples and apply a sample and hold D/A converter to sketch a reconstructed waveform.
For the following two problems the trigonometric identity

\[ \cos(a) \cos(b) = \frac{1}{2} \cos(a - b) + \frac{1}{2} \cos(a + b) \]

may be useful.

**Problem 4.** Suppose we receive the analog signal

\[ r_a(t) = A \cos(2\pi Ft + \theta). \]

Here the amplitude \( A \) is a constant but we do not know its value. We do know that the frequency \( F \) is 200 Hz and the phase \( \theta \) is \( \pi/4 \). We can follow the steps below to estimate the value of \( A \). Assume for purposes of calculation that the value of \( A \) is 10, i.e., use \( A = 10 \) in the above signal in your calculations.

**S1.** Multiply \( r_a(t) \) by \( x(t) \), where \( x(t) = \cos(2\pi 200t + \pi/4) \). Call the result \( y(t) \).

**S2.** Integrate \( y(t) \) from 0 to \( T \) and multiply the result by \( 2/T \). The result is your estimate of \( A \).

a. Follow the 2 steps above and estimate \( A \) using \( T = 6, 8, 16, 26, 126 \) msec.

b. Explain why following the 2 steps above will give the exact answer for \( A \) as \( T \to \infty \).

c. Determine (analytically) the finite values of \( T \) that will make your estimate for \( A \) exact and using the smallest such \( T \) follow the two steps above again to estimate \( A \).
Problem 5. Suppose we receive the analog signal

\[ r_a(t) = A \cos(2\pi 200t + \theta) \]

and sample it at 1000 Hz to get the digital signal

\[ r(n) = A \cos(0.4\pi n + \theta). \]

Here the amplitude \( A \) is a constant but we do \emph{not} know its value. Furthermore, we do \emph{not} know the \( \theta \) phase value. We can follow the steps below to estimate the value of \( A \). Assume for purposes of calculation that the value of \( A \) is 10 and \( \theta = \pi/4 \), i.e., use \( A = 10 \) and \( \theta = \pi/4 \) in the above signal in your calculations.

\textbf{S1.} Multiply \( r(n) \) by \( x_1(n) \) and \( x_2(n) \), where \( x_1(n) = \cos(0.4\pi n) \) and \( x_2(n) = \sin(0.4\pi n) \). Call the results \( y_1(n) \) and \( y_2(n) \), respectively.

\textbf{S2.} Simply add up the values of \( y_1(n) \) and \( y_2(n) \) for \( n = 0, 1, 2, \ldots N - 1 \) (some \( N \)) and take the average of each (divide by \( N \)) and then multiply the averages by 2. Call the results \( z_1 \) and \( z_2 \), respectively.

\textbf{S3.} Compute \( \sqrt{z_1^2 + z_2^2} \). This is the estimate of \( A \).

\begin{enumerate}
\item Follow the 3 steps above and estimate \( A \) using \( N = 5, 11, 19 \).
\item Explain why following the 3 steps above will give the exact answer for \( A \) as \( N \to \infty \).
\item Determine (analytically) the \emph{finite} values of \( N \) that will make your estimate for \( A \) exact and using the smallest such \( N \) follow the 3 steps above again to estimate \( A \).
\end{enumerate}
Problem 6. Use MATLAB for this problem. Let

\[ x_1(n) = u(n) - u(n - 2) \]
\[ x_2(n) = \delta(n) + \delta(n - 1) + u(n - 3) \]
\[ x_3(n) = \cos(2\pi n/15) \]
\[ x_4(n) = \exp(i2\pi n/20) \]
\[ x_5(n) = \sum_{k=0}^{n-1} u(k) \]

Plot each of these sequences on a separate graph for 0 \( \leq n \leq 40 \). For \( x_4(n) \) plot the real and imaginary parts separately.