Semiconductor laser is an important class of laser:

- Compact (\(\sim (100 \mu m)^3\) in size)
- Integrable (mature manufacturing technology)
- Low power consumption (\(\sim 10 \, mA, 2V\))
- High efficiency (\(\geq 50\%\))
- High modulation bandwidth (\(> 10 \, GHz\) due to short photon lifetime and carrier lifetime)
- Communication wavelengths (e.g. InGaAsP at 1.55\(\mu m\))
- High gain (\(> 10 \, cm^{-1}\) typical)
- Large longitudinal mode spacing (FSR = \(\frac{c}{\lambda} \frac{d}{d_{\text{small}}}\))
- Waveguide modes as transverse modes.
- Tunable wavelength (temperature/current can change the cavity length/index of refraction)
- Broad gain spectrum (\(> 10 \, THz\) due to energy bands)
In a semiconductor crystal, atoms are bound together closely. When we bring isolated atoms together, the electron clouds of individual atoms begin to overlap and the discrete energy levels broaden into energy bands.

Stimulated emission occurs when an input photon induces an electronic transition from the conduction (upper) band to the valence (lower) band.

Population inversion can be achieved by pumping a p-n junction electrically. By applying a forward bias voltage (V) greater than the band gap (Eg), lots of electrons from the n-side will diffuse into the p-side. Recombination of electron-hole pair releases a photon.

Laser oscillation requires a proper optical feedback. One easy way to implement the feedback is by cleaving the edge of the crystal. The surface reflection is usually sufficient for lasing.
I. Energy band

Energy of an electron depends on its state. Let the wavenumber of the electron be \( k \) then the momentum of the electron is \( \frac{\hbar}{2\pi} k \) 

\[ (\hbar = h / 2\pi) \]

Since energy \( E = \frac{1}{2} m^* (\text{momentum})^2 \), we have:

\[ (E_2 - E_1) = \frac{1}{2m^*} (\hbar k)^2 \quad (1) \]

\[ (E_r - E_i) = \frac{1}{2m^{*}\hbar^2} (\hbar k)^2 \quad (2) \]

\( m^*_e, m^*_h \) are called the effective mass of electron and hole, respectively. They are not equal to the free electron mass \( m_0 = 9.11 \times 10^{-31} \text{ kg} \) due to the presence of the crystal structure. For example, GaAs has \( m^*_e = 0.067 m_0, \ m^*_h = 0.52 m_0 \). (Holes are usually heavier.)
A.

Optical transition

Consider the absorption of a photon:

\[ |1\rangle + \text{photon} \rightarrow |2\rangle \]

(frequency \(\nu\) \(\rightarrow\) wavenumber \(k_p\))

Both energy and momentum must be conserved:

\[ E_2 = E_1 + h\nu - 0 \text{, and} \]

\[ k_2 = k_1 + h k_p \]

Since \( k = \frac{2\pi}{\lambda} \),

\[ k_p = \text{Wavelength of electron (\sim lattice constant)} \]
\[ k_{i,2} = \text{Wavelength of light (\sim infra-red)} \]
\[ \sim \frac{1\,\text{nm}}{1\,\mu\text{m}} = 10^{-3} \text{ typically} \]

\[ \Rightarrow \text{photon momentum } h k_p \text{ is usually neglected} \]

\[ \Rightarrow k_2 = k_1 (\approx k) \]

All optical transitions are represented as vertical lines in the \(E-k\) plots.

As a consequence, it is much easier to obtain population inversion in a direct gap material.

\( E \uparrow \)

\( \text{Direct gap. (GaAs)} \)

\( \downarrow k \)

\( \text{Indirect gap (Si, Ge)} \)
B Reduced effective mass.

Due to the conditions of energy and momentum conservation, the electron-hole pair that can take part in an optical transition is not arbitrary.

\[
(E_2 - E_c) = (E_1 - E_v) + h\nu - E_g
\]

\[
\frac{1}{2M_e^*} (\frac{\hbar k}{2})^2 = \frac{1}{2M_r^*} (\frac{\hbar k}{2})^2 + h\nu - E_g
\]

\[
\therefore h\nu - E_g = \frac{1}{2M_r^*} (\frac{\hbar k}{2})^2
\]

where \( m_r^* = \left( m_e^*^{-1} + m_h^*^{-1} \right)^{-1} \) is called the **reduced effective mass**.

Thus,

\[
\begin{align*}
E_2 - E_c &= \frac{m_r^*}{m_e^*} (h\nu - E_g) \\
E_v - E_1 &= \frac{m_r^*}{m_h^*} (h\nu - E_g)
\end{align*}
\]
Population of electrons/holes

We are then interested to find the population in the conduction/valence band. The population is the product of the density of states and the probability of occupancy.

Density of states \( [\, P(E)dE \ (m^{-3}) \,] \)

For a piece of semiconductor of volume \( L \times L \times L \), the \( k \)-vector of the wavefunction is quantized by the boundary conditions. For a given range of energy, \( E \) to \( E + dE \), there exist a number of state per volume, \( P(E)dE \).

\[ \frac{\hbar^2}{2m} \begin{pmatrix} \frac{\pi}{L} \hat{x} + \frac{\pi}{L} \hat{y} + \frac{\pi}{L} \hat{z} \end{pmatrix} \]

is quantized

\( (m, n, q \text{ are integers}) \)

\[ E = \frac{1}{2m} (\hbar k)^2 \]

\[ k = \frac{2\pi}{\hbar} \sqrt{2mE} \]

\[ dk = \frac{\pi}{\hbar} \sqrt{2mE} \ E^{-1/2} \ dE \]

In \( k \)-space, each state occupies \( (\pi/L)^3 \), so:

Density of states (in \( dE \)):

\[ P(E)dE = \left[ 2 \cdot \frac{4\pi k^2 dk}{8} \right] \frac{1}{L} \]

\( (m^{-3}) \)  

\( \hat{S} \)  

Spins \( N \) "Shell volume"  "Volume"  per mode
Substitution of \( k \) and \( dk \) gives:
\[
P(E) = \frac{4\pi (2m)^{3/2}}{h^3} E^{1/2} dE
\]

For conduction band,
\[
E \rightarrow E_c - E
m \rightarrow m_e^*
\] (see 1)

For valence band,
\[
E \rightarrow E_v - E
m \rightarrow m_v^*
\] (see 2)

\[
P_c(E_c) = \frac{4\pi (2m_e^*)^{3/2}}{h^3} \sqrt{E - E_c} \quad \cdots (6)
\]

\[
P(E_v) = \frac{4\pi (2m_v^*)^{3/2}}{h^3} \sqrt{E_v - E_v} \quad \cdots (7)
\]

For electron-hole pairs that can take part in optical transition of \( h\nu \):
\[
E \rightarrow h\nu - E_g
m \rightarrow m_r^*
\] (see 5)

The joint density of state of such electron-hole pair is thus:
\[
P_{jnt}(\nu) = \frac{4\pi (2m_r^*)^{3/2}}{h^3} \sqrt{h\nu - E_g} \quad \cdots (8)
\]

We can interpret \( P_{jnt}(\nu) d\nu \) as the number of available electron-hole paired states, that can be excited by optical or other means in \( \nu \rightarrow \nu + d\nu \) per volume.
B. Probability of occupation

The mere existence of a state does not guarantee that the state is occupied. For electrons, the probability for a state at energy $E$ being occupied is given by:

\[ f(E) = \frac{1}{e^{(E-E_F)/k_BT} + 1} \]  \hspace{1cm} (9)

which is called the Fermi-Dirac distribution. Here, $T$ is temperature, $k_B = 1.38 \times 10^{-23}$ J/K is the Boltzmann constant, and $E_F$ is a distribution parameter called the Fermi energy.

For a state with $E < E_F$, it has a high probability of being occupied, i.e., $f(E) > 1/2$.

For a state with $E > E_F$, it has a low probability of being occupied, i.e., $f(E) < 1/2$.

When a p-n junction is not biased, it is under thermal-equilibrium. The Fermi level for the conduction electrons, $E_{Fc}$, must lie the same as that for the valence electrons, $E_{Fv}$.

When a forward bias $V$ is applied, the Fermi levels are split:

\[ E_{Fc} - E_{Fv} = eV \]
The forward bias reduces the built-in potential barrier of a pn junction. Electrons diffuse from the n side to the p side, while holes diffuse from the p side to the n side. It creates electron-hole pairs that are ready to recombine and release a photon.

The electron-hole population can then be represented as follows:

\[ f_c(E) = \frac{1}{e^{(E - E_{Fc})/k_B T} + 1} \]

\[ f_v(E) = \frac{1}{e^{(E - E_{FV})/k_B T} + 1} \]
III Optical gain

We are finally in the position to find the optical gain of a semiconductor. The optical transition rates are still governed by the Einstein coefficients \(A_1, B_2, B_3\).

Although the gain can be derived from sketches, we can obtain the expression simply by incorporating the energy band into the well known formula:

\[
\text{Gain, } \gamma(v) = \frac{A_1 \lambda^2}{8 \pi n^2} \left[ N_2 - \frac{g}{g_2, N_1} \right] \\
\text{per m}^{-1} \left( \text{s}^{-1}(\text{m}^2) (\text{Hz})^{-1} (\text{m}^{-3}) \right)
\]

It was obtained in Chapter 7.4-7.5 for isolated atoms. The term \(g\left(N_2 - \frac{g}{g_2, N_1}\right)\) represents the inverted population capable of radiating at \(v\).

For a semiconductor, the term is replaced by:

\[
P_{\text{int}}(v) \left[ f_c(E_2) - f_v(E_1) \right] \\
\text{(m}^{-3} \text{Hz})^{-1}
\]

Density of state Inversion probability.

Hence, the gain (per unit length):

\[
\gamma(v) = \frac{A_1 \lambda^2}{8 \pi n^2} P_{\text{int}}(v) \left[ f_c(E_2) - f_v(E_1) \right]
\]
For an unbiased semiconductor, 
\[ f_{c}(E_2) \rightarrow 0 \quad f_{v}(E_1) \rightarrow 1 \]

\[ \gamma(v) = -\frac{A_2 \chi^2}{8 n_1 n_2} P_{\text{out}}(v) \]  
\therefore \text{Absorbs photons of } h\nu > E_g \]  
\[ \gamma(v) > 0 \iff f_{c}(E_2) > f_{v}(E_1) \]

\[ \frac{1}{e^{(E_2 - E_F)/k_B T} + 1} > \frac{1}{e^{(E_1 - E_F)/k_B T} + 1} \]
\[ e^{(E_1 - E_{F_F})/k_B T} > e^{(E_2 - E_{F_F})/k_B T} \]
\[ E_1 - E_{F_F} > E_2 - E_{F_F} \]
\[ \frac{E_{F_F} - E_{F_F}}{eV} > \frac{E_2 - E_1}{h\nu} \]

In other words, the population inversion condition is 
\[ eV = E_{F_F} - E_{F_F} > h\nu > E_g \]

\[ \text{Laser oscillation is thus possible by providing an optical feedback}. \]