We saw that when the longitudinal modes are all phase-locked together, a train of pulses separated by the round-trip transit time $T_{\text{round-trip}} = 2d/c$ is produced.

$$I(t) = \frac{E_0^2}{2\eta_0} \left[ \frac{\sin \left( \frac{N\omega_c t}{2} \right)}{\sin \left( \omega_c t \right)} \right]^2$$

How can we phase-lock the modes?

1. **Active Mode Locking**.

   ![Diagram of active mode locking](image)

   $\omega_0 + \omega_{ac} = \omega_s$
   
   $k_0 + k_{ac} = k_s$

   After millions of passes through the cavity, get a sharp train of pulses if RF frequency is $2 \left( \frac{\omega_0}{2\eta_0} \right)$.
Passive Mode-locking

Insert a very thin saturable absorber cell in the cavity

\[
\text{Transmission} \quad A \\
\begin{array}{c}
\text{Transmit} \\
\text{mum} = 1
\end{array}
\]

\[
0.1 \quad I_{\text{threshold}} \quad \text{Intensity}
\]

The thickness of the cell is \(< \text{ultimate pulsewidth} \sim 1/\Delta \lambda\)

As the photon density in the mode builds up there are

stochastic peaks in the intensity due to interference effects.

\[
\begin{array}{c}
\text{Stochastic peaks}
\end{array}
\]

\[
\text{time}
\]

If one of these peaks exceeds the intensity \(I_{\text{threshold}}\), then

the saturable absorber bleaches and the pulse is transmitted.

These photons establish the phase of the mode and begin

oscillate in the cavity. The recovery time of the absorber

\(< R_T \text{ transit time }\).
Q-switching.

This is a technique for obtaining higher peak power from pulsed lasers. It relies on suppressing the oscillation in the cavity by inserting an optical switch which is initially closed. Even though the initial small signal gain is many times the threshold gain, no oscillation is possible since there is no feedback. At some point the switch is rapidly opened whereupon the intensity rapidly builds up, saturating the gain and driving the inversion below the threshold value. What emerges is an intense (short, ns-range) laser pulse.

Diagram:

- Graph showing photon number \( \Phi_p \) as a function of time \( t \).
- Graph showing intensity as a function of time \( t \) with a sharp peak at \( t=0 \) when the switch is open.
How does the optical switch work?

Initially the polarizer and analyzer are crossed so that no photons are transmitted.

At \( t = 0 \) a voltage pulse of magnitude \( V_0 \) to rotate the polarizer by \( 90^\circ \) is applied. Now the analyzer transmits the photons and oscillation begins.

What are the boundary conditions?

\[ \phi_p = 0 \quad \text{at} \quad t = 0 \quad n_1 = N \]

when \( n = n_{th} \) \[ \phi_p = \phi_{p \text{ max}} \]

\( n = n_{final} \) \[ \phi_p = 0 \] again.