I. Spreading of an EM beam (Sec. 1.6 8.1.7)

Uncertainty principle

\[ \Delta w \Delta t \geq \frac{\hbar}{2} \Rightarrow \Delta E \Delta t \geq \frac{\hbar}{2} \text{ or } \frac{\hbar}{4\pi} \]

\[ \Delta k_x \Delta x \geq \frac{\hbar}{2} \]

\( \uparrow \)

Conjugate variables

Gaussian beam

\[ E(y) = E_0 \exp \left[ -\left( \frac{y}{w_0} \right)^2 \right] \]

Fourier transform.

\[ F(k_y) = \int_{-\infty}^{\infty} f(y) e^{ik_y y} \, dy. \]

So

\[ E(k_y) = \int_{-\infty}^{\infty} E_0 \exp \left[ -\left( \frac{y}{w_0} \right)^2 \right] e^{ik_y y} \, dy \]

\[ = E_0 \int_{-\infty}^{\infty} \exp \left[ -\left( \frac{y^2}{w_0^2} \right) + iky \right] \, dy. \]

\[ = E_0 \int_{-\infty}^{\infty} \exp \left[ -\frac{1}{w_0^2} \left( y^2 - ikyw_0^2 - \frac{1}{2} k^2 w_0^4 \right) \right] \exp \left( \frac{-kyw_0^2}{2} \right) \, dy \]

\[ = E_0 \int_{-\infty}^{\infty} \exp \left[ - \left( \frac{y - ikyw_0^2}{w_0} \right)^2 \right] \, dy \left( \exp \left( \frac{-kyw_0^2}{2} \right) \right) \]

\[ = E_0 \exp \left[ - \left( \frac{kyw_0^2}{2} \right)^2 \right] \int_{-\infty}^{\infty} \exp \left[ - \left( \frac{y - ikyw_0^2}{w_0} \right)^2 \right] \, dy \]
\[ \text{let } x = \frac{y - ikw_0^2}{2w_0} \]

\[ \Rightarrow dx = \frac{1}{w_0} \, dy \Rightarrow dy = w_0 \, dx \]

\[ E(k_y) = E_0 \exp \left[ -\left( \frac{kyw_0}{2} \right)^2 \right] \int_{-\infty}^{\infty} e^{-x^2} \cdot w_0 \, dx \]

\[ = E_0 w_0 \exp \left[ -\left( \frac{kyw_0}{2} \right)^2 \right] \int_{-\infty}^{\infty} e^{-x^2} \, dx \]

\[ = \sqrt{\pi} \cdot \frac{1}{w_0} \]

\[ \Rightarrow E(k_y) = E_0 \sqrt{\pi} w_0 \exp \left[ -\left( \frac{kyw_0}{2} \right)^2 \right] \]

\[ \text{Normalized} \]

\[ \text{E} \]

\[ w_0 \]

\[ \frac{1}{2} \]

\[ y \]

\[ \text{Normalized} \]

\[ \text{E} \]

\[ \frac{1}{2} \]

\[ k_y \]

\[ \frac{1}{w_0} \]

\[ \text{Note:} \]

\[ \int_{-\infty}^{\infty} e^{-x^2} \, dx \]

\[ = \int \int_{-\infty}^{\infty} e^{-x^2} \, dx \cdot \int_{-\infty}^{\infty} e^{-y^2} \, dy \]

\[ = \int \int_{-\infty}^{\infty} e^{-(x^2+y^2)} \, dx \, dy \]

\[ = \int \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} e^{-r^2} \, r \, dr \, d\theta \]

\[ = \int_{\theta=0}^{\pi} \left[ \frac{1}{2} e^{-r^2} \right]_{r=0}^{r=\infty} \, d\theta \]

\[ = \sqrt{\pi} \]
\[ \frac{\theta_0}{2} \approx \tan \frac{\theta_0}{2} = \frac{\Delta k_y}{k_z} = \frac{2}{w_0} \cdot \frac{\lambda}{2\pi} = \frac{\lambda}{w_0\pi} \]

\[ \theta_0 = \frac{2\lambda}{w_0\pi} \]  
Beam spread angle

\[ \theta_0 \propto \lambda \text{ and } \theta_0 \propto \frac{1}{w_0} \]

II. Problem 1.5

<table>
<thead>
<tr>
<th>Source</th>
<th>eV</th>
<th>(\lambda (\text{Å}))</th>
<th>(\lambda (\text{nm}))</th>
<th>(\nu (\text{Hz}))</th>
<th>(\overline{\nu} (\text{cm}^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>GaAs</td>
<td>1.47</td>
<td></td>
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</tr>
</tbody>
</table>

photon energy =  \(h \nu = 6.626 \times 10^{-34} \times \nu \)  
\((\text{J} \cdot \text{s})\)  
\((\text{s}^{-1})\)

\[ \nu = 4.136 \times 10^{15} \times \nu \]  
\((\text{eV} \cdot \text{s})\)  
\((\text{s}^{-1})\)

\[ \nu = \frac{1.47}{4.136 \times 10^{15}} = 3.554 \times 10^{14} \text{ (Hz)} \]

\[ \lambda = \frac{c}{\nu} = \frac{2 \times 10^8}{3.554 \times 10^{14}} = 8.441 \times 10^{-7} \text{ (m)} \]

\[ = 8.441 \times 10^3 \text{ (Å)} \]

\[ = 8.441 \times 10^2 \text{ (nm)} \]

\[ \overline{\nu} = \frac{1}{\lambda} = \frac{1}{8.441 \times 10^{-7} \times 10^2} = 1.18 \times 10^4 \text{ (cm}^{-1}) \]
A sinusoidal wave in time

\[ f(t) = \cos(\omega t) \]

phase \( \phi \)

\[ T \]

\[ 0 \]

\[ 2\pi \]

\[ \omega = \frac{2\pi}{T} \]

A sinusoidal wave in space

\[ f(z) = \cos(\kappa z) \]

phase \( \phi \)

\[ \lambda \]

\[ 0 \]

\[ 2\pi \]

\[ k = \frac{2\pi}{\lambda} \]

An EM wave is a sinusoidal in space and time

\[ E(\mathbf{r}, t) = E_0 \cos \left( \omega t - \mathbf{k} \cdot \mathbf{r} \right) \]

amplitude phase

\[ = \text{Re} \left\{ E_0 \exp \left[ j(\omega t - \mathbf{k} \cdot \mathbf{r}) \right] \right\} \]

\[ = \frac{E_0}{2} \left\{ \exp \left[ j(\omega t - \mathbf{k} \cdot \mathbf{r}) \right] + \text{c.c.} \right\} \]

"Wavefront" is the surface defined by the locus of points that have constant phase. In free space, \( |\mathbf{k}| = \frac{2\pi}{\lambda} = \frac{\omega}{c} \)

Direction of \( \mathbf{k} \) is \( \perp \) to wavefronts
Michelson interferometer

\[ E_1 = \frac{E_0}{jz} \exp \left[ -j \left( k \cos \frac{\theta}{2} \frac{z}{2} + k \sin \frac{\theta}{2} \frac{x}{2} \right) \right] \exp(-jzL_1) \exp(-j\Delta \phi) \]

\[ E_2 = \frac{E_0}{jz} \exp \left[ -j \left( k \cos \frac{\theta}{2} \frac{z}{2} - k \sin \frac{\theta}{2} \frac{x}{2} \right) \right] \exp(-jzL_2) \]

\[ I = 2 \left( \frac{E_0^2}{220} \right) \cos^2 \left[ \frac{k \Theta x}{2} + k (L_2 - L_1) + \frac{\Delta \phi}{2} \right] \]

If \( \Delta \phi \) is constant, there are clear bright and dark fringes.

If \( \frac{d\Delta \phi}{dt} \) is large, the fringes would get averaged out and become less visible.

If \( \frac{d\Delta \phi}{dt} = 10^{-5} \omega = 10^{-5} \frac{2\pi c}{\lambda} \)

For \( \frac{\Delta \phi}{2} = \frac{\pi}{2} \Rightarrow \Delta \phi = \frac{d\phi}{dt} \cdot \text{coherence time} = 10^{-5} \frac{2\pi c}{\lambda} \times \frac{2(L_2 - L_1)}{c} = \pi \)

\( \Rightarrow L_2 - L_1 = 10^5 \times \frac{\lambda}{4} \) coherence length