EE 172L
Diffraction and Gaussian Beams

**Purpose:** Compare the measured diffraction patterns and Gaussian beam propagation with theoretical predictions.

**Background:** The reference textbook Guenther provides an excellent overview on the subjects of diffraction and Gaussian beams. You should pay particular attention to the following concepts:

1. The difference between Fresnel and Fraunhofer diffraction formulations and their limitations. In this Lab, we will use Fraunhofer diffraction only because the observation distance from the diffraction aperture is much larger than the size of the aperture and the wavelength.
2. Through the use of far-field approximation, the Fraunhofer diffraction field equals the two-dimensional spatial Fourier transform of the aperture’s transmission function.
3. The meaning of spatial frequency.
4. The far-field diffracted pattern from a single rectangular, double rectangle and circular aperture. Where does the intensity min. and max. occur?
5. The definition for the size of a Gaussian beam.
6. The propagation equation of a Gaussian beam.

**PreLab Reading:** “Modern Optics” by R. Guenther: Huygen’s Principle and Fresnel Formulation (p323-331), Fraunhofer diffraction (p361-366, 369-379, 386-387), Gaussian beam (p336-343, 349-350), 2-D Fourier Transform (p241-245), “Laser Electronics: by Verdeyen, Chap. 3 (3.3.10) (3.3.7).

**Notation Conversion:**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Guenther</th>
<th>Verdeyen</th>
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<tbody>
<tr>
<td>Beam expansion angle</td>
<td>$\theta = \frac{\lambda_0}{\pi , n , \omega_o}$</td>
<td>$\theta = \frac{2\lambda_0}{\pi , n , W_o}$</td>
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<tr>
<td>Confocal parameter</td>
<td>$q_o = \frac{\pi , n , \omega_o^2}{\lambda_o}$</td>
<td>$2Z_o = \frac{2 , \pi , n , W_o^2}{\lambda_o}$</td>
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PreLab Exercise:
1. Sketch the diffraction pattern for a double slit using $\lambda = 0.5 \mu m$ at the distance of 10m. Both slits are 0.1 mm wide and 0.6mm between the centers of the two slits.
2. Calculate the ratio of peak intensity between the center ($0^{th}$), 1$^{st}$, 2$^{nd}$ and 3$^{rd}$ lobe in a single slit diffraction pattern based on the expression given by equation 10-20 in Guenther. We will assume that the slit is $\infty$ long in the y direction ($y_0 = \infty$). Therefore, it’s just a 1-dimension problem.
3. Prove that a light beam with a Gaussian intensity profile will transform into another Gaussian as it propagates. To simplify the math, deal with only the Gaussian portion and in one dimension only, since a Gaussian beam is circular symmetry; i.e., write the integral as:
\[
\int_{-\infty}^{\infty} \exp[-(x/\omega_o)^2] \exp(ik_z x) dx
\]
4. Presume that I obstruct a Gaussian beam with a circular aperture. Calculate the energy remaining in the beam after it passes through the aperture as a function of beam waist and aperture radius. What percentage of energy is remaining if the aperture radius equals the beam waist?
5. A fundamental Gaussian (TEM$_{\infty}$ Mode) beam propagates along the z direction. Given the wavelength $\lambda$, the beam size ($\omega_1, \omega_2$) measured at two different distances ($Z_1, Z_2$) from a reference point that is at an unknown distance $\Delta$ from the beam waist. Derive the expression for the confocal parameter $q_o$ and the location of the beam waist $\Delta$ in terms of the given values.
\[
q_o = \frac{\pi \omega_o^2}{\lambda}, \quad \omega^2(Z) = \omega_o^2 \left[ 1 + \left( \frac{Z}{q_o} \right)^2 \right]
\]
Note: Do not write out the full expression. Just work out to the point where it’s clear that the solution can be obtained numerically.
Experimental Procedures:

1. Measure the camera’s horizontal field of view by taking the image of a calibration ruler. Measure the distance from the slit to the image plane.

2. Measure the far field pattern from a single slit for two different slit widths. From the measured diffraction patterns calculate the width of each slit. Compare the relative intensity of each lobe with the calculation made in the pre-lab.

   Note: Use only the center portion of the laser beam to illuminate the slit. Because the wavefront of a Gaussian beam is nearly flat at the center, it provides a close approximation of plane wave illumination. (the wavelength of the red HeNe laser is 632.8 nm)

3. Repeat part 2 with a double slit. Calculate the slit width and the distance between slits.

4. Repeat part 2 with a circular pinhole. Calculate the diameter of the pinhole. Note that the expression of the diffraction pattern contains the first order Bessel function \( J_1 \). A plot of Bessel functions is given at the end of this hand-out.

5. Assume the laser beam is a fundamental Gaussian and the beam waist is located at the output window of the laser. Measure the beam size at 3 different distances from the laser and compute the beam waist size, \( \omega_w \), and the confocal beam parameter, \( q_c \). Keep in mind that the camera responds to intensity, not electric field. Beam size, \( \omega \), is defined as from the center of the beam to where the intensity is down by a factor of \( e^{-2} \). You can use the formulations from pre-lab Prob. 5 with \( \Delta = 0 \).

Additional Information on Equipment:

Obtaining optimum image data using the CCD camera:

**CCD-Charged Coupled Device.**

**Focusing** – Because the depth of focus is small when the aperture (f stop) is wide open, the image would be very sensitive to misadjust, making it easier to locate the focus position. First, turn down ambient light and open up the aperture. Then adjust the focus to obtain a sharp image.

**Intensity saturation** – If the light intensity exceeds the upper limit of the CCD element, the digitized value will be fixed at the upper limit (saturation). The intensity information is lost at that location. Therefore, it’s VERY important to ensure that the entire image is acquired without any pixel being saturated. Saturation can be detected by looking at the intensity value using the line-profile function provided by the software.

**Cutting Out An Image** - Diffraction patterns from long vertical slits are nearly vertically independent. Therefore, using the software, most of the visible portion of the image can be cut out and vertically summed up to improve the intensity resolution of the pattern. However, circular diffraction patterns are NOT vertically independent. Therefore, only a small portion of the center should be cut out and used for intensity profile.
Large Intensity Variation – Most of the diffraction patterns consist of very bright center lobe and very dim side lobes. Since the dynamic range of the digitization is limited to 8-bit (0 to 255), it’s impossible to resolve detail structure of all parts of the pattern with one exposure setting. However, using multiple exposures, all parts of the pattern can be obtained. Take the first image with no saturation of the center lobe. This allows the determination of intensity ratio between the center and the first lobe. Take the second image by opening up the aperture until just before the first lobe becomes saturated and details of the higher order lobes become visible. This allows the intensity ratio between the first lobe and the higher order lobes to be determined. Combining the information from the two images, with the proper scaling factor, the entire diffraction pattern can be constructed.

Speckle Remover – Because laser light is highly monochromatic when reflected from a surface that’s not perfectly smooth, interference patterns (speckles) will form between neighboring high and low points on the surface, giving reflected laser light the characteristic “sparkling” look. Speckles add undesirable intensity modulation on the image. To get around this problem, the diffraction pattern is projected onto a rotating board, which continuously changes the speckle pattern by moving the reflection surface. Because the camera takes a finite time to acquire the image, speckles will “wash out” due to statistical averaging, resulting in a speckle-free, and smooth image pattern.