Wireless Communication Links and Antennas

EE 162A

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OUTLINE

• Electrically small dipole

• Half wavelength dipole

• Monopole

• Small loop
Finite Length Dipole: Current Distribution

Current Density: \( \vec{J}(\vec{r}') = I(z') \delta(x') \delta(y') \hat{z} = \frac{A}{m^2} \) diode along Z axis

A good approximation for the current:
\[
I(z') = I_m \sin\left[\beta\left(\frac{l}{2} - |z'|\right)\right]
\]
diode length

Note:
- Current goes to zero at the ends
- For different length dipoles, current distribution takes various distributions

For far field computation:
\[
\vec{A} = \mu \frac{e^{-j\beta r}}{4\pi r} \int_{V} \vec{J}(\vec{r}') e^{j\beta \vec{r}' \cdot \hat{r}} \, dv' = \mu \frac{e^{-j\beta r}}{4\pi r} \int_{-l/2}^{l/2} I(z') e^{j\beta \vec{r}' \cdot \hat{r}} \, dz'
\]
Electrically Small Dipole

Example: AM broadcast receiver antenna

$|l < \lambda/10|$ Electrically small, physical large at low frequency

Current distribution: $I(z') = I_m \sin\left(\frac{\beta}{2} - |z'|\right) \approx I_A (1 - 2|z'|/l)$

Radiation resistance: $R_r = 20\pi^2 \left(\frac{\Delta z}{\lambda}\right)^2 = \frac{1}{4} R_{r \text{ideal}}$

To increase the radiation resistance, one tries to obtain a uniform current on the dipole. Two effective approaches are capacitor-plate antenna and transmission line loaded antenna.
Half Wavelength Dipole (1)

Among various length dipoles, half-wavelength (λ/2) dipoles are the most popular ones. This is because of their “good” input impedance.

\[ \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \Theta \hat{z} \]

Recall for \( \hat{z} \) oriented linear current \( \hat{r} \cdot \hat{r}' = z' \cos \theta \)

E-field:

\[ E_{\theta} = j \omega A_{\theta} = j \omega \sin \theta A_{z} = j \omega \mu \sin \theta \frac{e^{-j \beta r}}{4 \pi r} \int I(z') e^{j \beta z' \cos \theta} \, dz' \]

For \( l = \frac{\lambda}{2} \)

\[ f_{un} = I_m \int_{-\lambda/4}^{0} \sin(\frac{\pi}{2} + \beta z') e^{j \beta z' \cos \theta} \, dz' \]

\[ + I_m \int_{0}^{\lambda/4} \sin(\frac{\pi}{2} - \beta z') e^{j \beta z' \cos \theta} \, dz' \]

We need to evaluate this integral!!
Half Wavelength Dipole (2)

Integral Identity:
\[ \int \sin(a + bx)e^{cx} \, dx = \frac{e^{cx}}{b^2 + c^2} \left[ c \sin(a + bx) - b \cos(a + bx) \right] \]

Then after some manipulation:
\[ f_{un} = \frac{I_m}{\beta \sin^2 \theta} 2 \cos \left( \frac{\pi}{2} \cos \theta \right) \]

Finally:
\[ E_\theta = j \omega \mu \frac{2I_m}{\beta} e^{-jkr} \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{4\pi r} \frac{\sin \theta}{\sin^2 \theta} \]

Pattern:
\[ F(\theta) = \sin \theta \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin^2 \theta} \]

Note:
\[ \text{at } \theta = \frac{\pi}{2}; \quad F \left( \frac{\pi}{2} \right) = 1 \]

Pattern factor:
radiation pattern

Element pattern:

Each differential current element radiates \[ \sin \theta \]
Characteristics of dipole antennas

<table>
<thead>
<tr>
<th>Dipole Type</th>
<th>Length</th>
<th>Current</th>
<th>Pattern</th>
<th>HP</th>
<th>D (dB)</th>
<th>( R_s ) (Ω)</th>
<th>( R_{ohmic} ) (Ω)</th>
<th>Current Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ideal</td>
<td>( L \ll \lambda )</td>
<td>Uniform</td>
<td>( \sin \theta )</td>
<td>90°</td>
<td>1.5</td>
<td>1.76</td>
<td>( \frac{R_s L}{2 \pi a} )</td>
<td>( l_m )</td>
</tr>
<tr>
<td>Short</td>
<td>( L \ll \lambda )</td>
<td>Triangle</td>
<td>( \sin \theta )</td>
<td>90°</td>
<td>1.5</td>
<td>1.76</td>
<td>( \frac{2 \pi}{2} )</td>
<td>( \frac{R_s L}{2 \pi a} )</td>
</tr>
<tr>
<td>Half-wave</td>
<td>( L = 0.5 \lambda )</td>
<td>Sinusoid</td>
<td>( \frac{\cos \left( \frac{\pi}{2} \cos \theta \right)}{\sin \theta} )</td>
<td>78°</td>
<td>1.64</td>
<td>2.15</td>
<td>( \approx 70 )</td>
<td>( \frac{R_s \lambda}{2 \pi a} )</td>
</tr>
</tbody>
</table>

Note: For an antenna of length \( L \), that carries an axially uniform current:

\[
R_{ohmic} \approx \frac{L}{2 \pi a} R_s \quad \Rightarrow \quad \text{Surface resistance, } R_s = \sqrt{\frac{\omega \mu}{2 \sigma}}; \quad \omega = 2\pi f
\]

General:

\[
R_{ohmic} = \frac{1}{|I_A|^2} \frac{R_s}{2\pi a} \int_{-l/2}^{l/2} |I_z|^2 dz
\]
Antennas above a Ground Plane

- In many practical situations, antennas are mounted on ground planes.

- In practice, ground planes are metallic finite size and may not be planar.

- A useful approximation that allows the application of image theory is to assume that the ground plane is “infinite” in extend and perfectly conducting (P.E.C)
Image Theory

Perfect Electric Conductor (PEC)

\[ \sigma = \infty \text{ (Electric conductor)} \]

Perfect Magnetic Conductor (PMC)

\[ h \]

Actual sources

Images
Monopole Antennas

- Image theory allows one to create equivalent known problem

Infinite ground plane

- In the upper hemisphere, monopole creates exactly the same electromagnetic fields as the dipole

Ideal monopole

\[
P_{\text{mono}} = \frac{1}{2} P_{\text{dipole}}
\]

\[
D_{\text{mono}} = \frac{4\pi}{\Omega_{A,\text{mono}}} = \frac{4\pi}{\frac{1}{2} \Omega_{A,\text{dipole}}} = 2D_{\text{dipole}}
\]

Directivity of a monopole is twice of dipole

\[
R_{r,\text{mono}} = \frac{1}{2} R_{r,\text{dipole}} = 40\pi^2 \left(\frac{h}{\lambda}\right)^2
\]

Length of short monopole
Small Loop Antennas

- Small loop antennas:

  \[ \mathcal{A} = \mu \frac{e^{-j\beta r}}{4\pi r} \int_{V'} \vec{J} e^{j\beta \vec{r} \cdot \hat{r}} \, dv' = \mu \frac{e^{-j\beta r}}{4\pi r} \int_{C} \vec{I} e^{j\beta \vec{r} \cdot \hat{r}} \, dc' \]

  Perimeter \( << \lambda \), **UNIFORM** time varying current

It turns out that the radiation fields of small loops are independent of the shape of the loop and depend only on the area of the loop.

Note: It appears to be easier to construct \( \mathbf{A} \) for the square loop.
Small Loop Antennas: Square Loop

Small square loop can be thought of as combination of “4” ideal dipoles.

\[ \vec{A}_{\text{small square}} = \hat{x}A_1 + \hat{y}A_2 + \hat{x}A_3 + \hat{y}A_4 \]

where

\[
\begin{align*}
A_x &= \frac{\mu Il}{4\pi} \left( \frac{e^{-j\beta R_1}}{R_1} - \frac{e^{-j\beta R_3}}{R_3} \right) \\
A_y &= \frac{\mu Il}{4\pi} \left( \frac{e^{-j\beta R_2}}{R_2} - \frac{e^{-j\beta R_4}}{R_4} \right)
\end{align*}
\]

Far field approximation:

\[
\begin{align*}
R_1 &= r + \frac{l}{2} \sin \theta \sin \phi \\
R_3 &= r - \frac{l}{2} \sin \theta \sin \phi \\
R_2 &= r - \frac{l}{2} \sin \theta \cos \phi \\
R_4 &= r + \frac{l}{2} \sin \theta \cos \phi
\end{align*}
\]

for amplitude term:

\[ R_1 \approx R_2 \approx R_3 \approx R_4 \approx R \]
Small Loop Antennas: Square Loop

Then: \[ A_x = \frac{\mu Il e^{-j\beta r}}{4\pi r} \left[ e^{-j\beta(l/2)\sin^2\theta\sin^2\phi} - e^{j\beta(l/2)\sin^2\theta\sin^2\phi} \right] \]

Recall: \[ \sin \alpha = \frac{1}{2j}(e^{j\alpha} - e^{-j\alpha}) \]

Finally:
\[
\begin{align*}
A_x &= -2j \frac{\mu Il e^{-j\beta r}}{4\pi r} \sin \left( \frac{\beta l}{2} \sin \theta \sin \phi \right) \\
A_y &= 2j \frac{\mu Il e^{-j\beta r}}{4\pi r} \sin \left( \frac{\beta l}{2} \sin \theta \cos \phi \right)
\end{align*}
\]

or: \[ \vec{A} = A_x \hat{x} + A_y \hat{y} = j\beta l^2 \frac{\mu Il e^{-j\beta r}}{4\pi r} \sin \theta (\sin \phi \hat{x} + \cos \phi \hat{y}) \]

\[ \vec{E} \text{ & } \vec{H} \text{ fields: } \vec{E} = -j \omega \vec{A} \Rightarrow \]

Note: \[ S = l^2 \text{ (area of loop)} \]
\[ \omega \mu \beta^2 = \eta \beta^2 \]

\[ \vec{E} = \eta \beta^2 S \frac{I e^{-j\beta r}}{4\pi r} \sin \theta \phi \]
\[ \vec{H} = \frac{1}{\eta} \hat{r} \times \vec{E} = -\beta^2 S \frac{I e^{-j\beta r}}{4\pi r} \sin \theta \hat{\phi} \]
Small Loop Vs. Ideal Dipole

Define $I_m \Delta z = j \omega \mu I S$

![Diagram of small loop and ideal dipole](image)

**Small Loop in x-y Plane**

- Loop area $S$
- $I$ current
- Ideal magnetic dipole along z axis

**Equations**

- $E = - j b I_m D z \frac{e^{-jbr}}{4 pr} \sin q \hat{r}$
- $H = j \omega e I_m D z \frac{e^{-jbr}}{4 pr} \sin q \hat{q}$
- $\lambda = 90^\circ$
- $W_A = \frac{8 \pi}{3}$
- $D = \frac{3}{2}$ or 1.75dB

- $R_r = 4 p^2 \times 80 p^2 \frac{\alpha S \delta^2}{\xi} \hat{r} \hat{\phi}$ ohm

**Ideal Dipole along z axis**

- $E = j \omega \mu I D z \frac{e^{-jbr}}{4 pr} \sin \theta \hat{r}$
- $H = j \beta I D z \frac{e^{-jbr}}{4 pr} \sin \theta \hat{\phi}$
- $\lambda = 90^\circ$
- $\Omega_A = \frac{8 \pi}{3}$
- $D = \frac{3}{2}$ or 1.75dB

- $R_r = 80 \pi^2 \left( \frac{\Delta z}{\lambda} \right)^2$ ohm
Duality Theorem

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<thead>
<tr>
<th>Dual equations for electric (J) and magnetic (M) current sources</th>
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</tr>
<tr>
<td><strong>Magnetic sources (J = 0, M ≠ 0)</strong></td>
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<tr>
<td>( \nabla \times E_A = -j\omega \mu H_A )</td>
</tr>
<tr>
<td>( \nabla \times H_A = J + j\omega \varepsilon E_A )</td>
</tr>
<tr>
<td>( \nabla^2 A + \beta^2 A = -\mu J )</td>
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<tr>
<td>( A = \frac{\mu}{4\pi} \iiint_V \frac{J e^{-jrR}}{r} , dv' )</td>
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<tr>
<td>( H_A = \frac{1}{\mu} \nabla \times A )</td>
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<tr>
<td>( E_A = -j\omega A - j\frac{1}{\omega \mu \varepsilon} \nabla (\nabla \cdot A) )</td>
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<tr>
<td>( \nabla \times H_F = j\omega \varepsilon E_F )</td>
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<tr>
<td>( -\nabla \times E_F = M + j\omega \mu H_F )</td>
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<tr>
<td>( \nabla^2 F + \beta^2 F = -\varepsilon M )</td>
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<tr>
<td>( F = \frac{\varepsilon}{4\pi} \iiint_V \frac{M e^{-jrR}}{r} , dv' )</td>
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<td>( E_F = -\frac{1}{\varepsilon} \nabla \times F )</td>
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