Wireless Communication Links and Antennas

EE 162A

Spring 2007, UCLA

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OUTLINE

• Input impedance

• Radiation resistance

• Plane wave and polarization
Typical Antenna Parameters

CIRCUIT QUANTITIES
- Radiation resistance, $R_r$
- Antenna temperature, $T_A$

ANTENNA QUANTITIES
- Field patterns $E_\theta (\theta, \phi)$, $E_\phi (\theta, \phi)$, $\delta (\theta, \phi)$
- Power pattern $P_p (\theta, \phi)$
- Beam area, $\Omega_A$
- Directivity, $D$
- Gain, $G$
- Effective aperture, $A$
- Radar cross section, $\sigma$

SPACE QUANTITIES
- Transition device
- Circuit element
- Impedance $Z$
Input Impedance of Antennas (1)

- We need signal source generator to sustain the oscillation of the current on the antenna.
- **Input impedance** is defined as the impedance presented by an antenna at its terminals.

Equivalent Circuits:

\[ Z_g = R_A + jX_A \]

\[ Z_A = R_A + jX_A \]

radiation resistance

radiated waves
Input Impedance of Antennas (2)

\[ I_g = \frac{V_g}{Z_A + Z_g} = \frac{V_g}{(R_r + R_{ohmic} + R_g) + j(X_A + X_g)} \]

Then

\[ P_r = \frac{1}{2} |I_g|^2 R_r = \frac{|V_g|^2}{2} \left[ \frac{R_r}{(R_r + R_{ohmic} + R_g)^2 + (X_A + X_g)^2} \right] \]

Power delivered to the antenna for radiation

\[ P_L = \frac{1}{2} |I_g|^2 R_{ohmic} = \frac{|V_g|^2}{2} \left[ \frac{R_{ohmic}}{(R_r + R_{ohmic} + R_g)^2 + (X_A + X_g)^2} \right] \]

Power dissipated as heat on the antenna

\[ P_g = \frac{1}{2} |I_g|^2 R_g = \frac{|V_g|^2}{2} \left[ \frac{R_g}{(R_r + R_{ohmic} + R_g)^2 + (X_A + X_g)^2} \right] \]

Power dissipated as heat on the internal resistance of the generator

As you know from circuit courses, the maximum power is delivered to the antenna when there is a **conjugate match**.

\[ Z_A = Z_g^* \]

\[ R_r + R_{ohmic} = R_g \]

\[ X_A = -X_g \]
Under conjugate match situation:

\[ P_r = \frac{|V_g|^2 R_r}{8 (R_r + R_{\text{ohmic}})^2}, \text{W} \]

\[ P_L = \frac{|V_g|^2 R_{\text{ohmic}}}{8 (R_r + R_{\text{ohmic}})^2}, \text{W} \]

\[ P_g = \frac{|V_g|^2 R_g}{8 (R_r + R_{\text{ohmic}})^2} = \frac{|V_g|^2}{8 R_g}, \text{W} \]

It is clear that: \[ P_g = P_r + P_L \]

Power supplied by the generator:

\[ P_g = \frac{1}{2} V_g I_g^* = \frac{|V_g|^2}{4} \frac{1}{R_r + R_L}, \text{W} \]

**Observation:** Under conjugate match condition, of the power that is supplied by the generator, half is dissipated as heat in the internal resistance \( R_g \) and the other half is delivered to the antenna.
Antenna Radiation Efficiency

\[ e_r (\text{or } \eta_r) \equiv \frac{\text{Power radiated by the antenna}}{\text{Power delivered to the antenna}} = \frac{P_r}{P_r + P_{\text{ohmic}}} \]

then

\[ e_r = \frac{1}{2} \frac{R_r |I_g|^2}{\frac{1}{2} R_r |I_g|^2 + \frac{1}{2} R_{\text{ohmic}} |I_g|^2} = \frac{R_r}{R_r + R_{\text{ohmic}}} = \frac{R_r}{R_A} \]

For many antennas, this radiation efficiency is nearly 100%.

For small antennas in terms of the wavelength, this radiation efficiency can be low. This means that the antenna is not an effective radiation of electromagnetic wave.
Radiation Resistance of an Ideal Dipole

Recall: \[ P_r = \frac{1}{2} |I_g|^2 R_r \Rightarrow R_r = \frac{2}{|I_g|^2} P_r \] \rightarrow \text{radiated power}

Recall: \[ P_r = \frac{1}{2} \Re \int \int (\mathbf{E} \times \mathbf{H}^*) \cdot d\mathbf{s} \]

Also recall: \[ P_r = \frac{1}{2\eta} \int \int_{0}^{2\pi} \int_{0}^{\pi} (|E_\theta|^2 + |E_\phi|^2) r^2 \sin \theta d\theta d\phi \]

(1) Write the far field: \[ \vec{E} = j \omega \mu I_g \Delta z \frac{e^{-j\beta r}}{4\pi r} \sin \theta \hat{\theta} \]

(2) \[ P_r = \frac{1}{2\eta} (\omega \mu I_g \Delta z)^2 \frac{1}{16\pi^2} \int \int_{0}^{2\pi} \int_{0}^{\pi} \sin^3 \theta d\theta d\phi = \frac{\omega \mu \beta}{12\pi} (I_g \Delta z)^2 \]

Finally: \[ R_r = \frac{2}{|I_g|^2} P_r = 80\pi^2 \left( \frac{\Delta z}{\lambda} \right)^2 \]

Since \( \Delta z \ll \lambda \) for an ideal dipole, \( R_r \) is very small.
Uniform Plane Waves (1)

Maxwell’s equations: lossless, source free, time harmonic

\[ \nabla \times \vec{E} = -j\omega \vec{B} \quad \nabla \cdot \vec{D} = 0 \]
\[ \nabla \times \vec{H} = j\omega \vec{D} \quad \nabla \cdot \vec{B} = 0 \]

Assume plane wave behavior → all field \( \propto e^{-j\vec{\beta} \cdot \vec{r}} \)

Note: \( \vec{\beta} \cdot \vec{r} = (\beta_x \hat{x} + \beta_y \hat{y} + \beta_z \hat{z}) \cdot (x\hat{x} + y\hat{y} + z\hat{z}) \)

Perform spatial differention, one finds:

\[ \nabla \rightarrow -j\vec{\beta} \]
Uniform Plane Waves (2)

Maxwell’s equations become:

\[
\begin{align*}
- j\vec{\beta} \times \vec{E} &= - j\omega \vec{B} \\
- j\vec{\beta} \times \vec{H} &= j\omega \vec{D} \\
- j\vec{\beta} \cdot \vec{D} &= 0 \\
- j\vec{\beta} \cdot \vec{B} &= 0
\end{align*}
\]

Important observation: Even in complex media where

\[
\vec{D} \neq \varepsilon \vec{E}, \quad \text{and} \quad \vec{B} \neq \mu \vec{H}
\]

Both \(\vec{D}\) and \(\vec{B}\) are orthogonal to \(\vec{\beta}\). This leads to the KDB (K used for \(\beta\)) system for plane wave analysis in complex media.
Uniform Plane Waves (3)

Let us consider simple media for now:

\[ \vec{D} = \varepsilon\vec{E}; \quad \vec{B} = \mu\vec{H} \quad \varepsilon, \mu \text{ are scalars} \]

Then:

\[ \begin{align*}
\vec{\beta} \times \vec{E} &= \omega\mu\vec{H} \\
-\vec{\beta} \times \vec{H} &= \omega\varepsilon\vec{E} \\
\vec{\beta} \cdot \vec{E} &= 0 \\
\vec{\beta} \cdot \vec{H} &= 0
\end{align*} \]

Important result: \((\vec{\beta}, \vec{E}, \vec{H})\) create orthogonal vectors.
For plane waves, the wave equation becomes:

\[ \vec{H} = \frac{1}{\omega \mu} \vec{\beta} \times \vec{E} \]

\[ \vec{\beta} \times (\vec{\beta} \times \vec{E}) = \omega \mu (\vec{\beta} \times \vec{H}) = \omega \mu (-\omega \epsilon \vec{E}) \]

\[ (\vec{\beta} \cdot \vec{E}) \vec{\beta} - (\vec{\beta} \cdot \vec{\beta}) \vec{E} = -\omega^2 \mu \epsilon \vec{E} \]

\[ (\vec{\beta} \cdot \vec{\beta}) \vec{E} = \omega^2 \mu \epsilon \vec{E} \]

\[ (\vec{\beta} \cdot \vec{\beta}) = \omega^2 \mu \epsilon \]

\[ \beta_x^2 + \beta_y^2 + \beta_z^2 = \omega^2 \mu \epsilon = k^2 \]

Note: \( \beta_z \) is not independent of \( \beta_x \) and \( \beta_y \).  
\[ |\vec{H}| = \sqrt{\frac{\epsilon}{\mu}} |\vec{E}| = |\vec{E}| / \eta \]
Polarization (1)

For a plane wave propagating in z direction

Phasor: \[ \mathbf{E} = (\hat{x}E_x + \hat{y}E_y)e^{-j\beta_z z} = (\hat{x} | E_x e^{j\phi_x} + \hat{y} | E_y e^{j\phi_y})e^{-j\beta_z z} \]

Instantaneous field:

\[ \mathbf{E} = \text{Re}[\mathbf{E}e^{j\omega t}] = \hat{x} | E_x \cos(\omega t - \beta_z z + \phi_x) + \hat{y} | E_y \cos(\omega t - \beta_z z + \phi_y) \]

Depending on the values of \( \phi_x \) and \( \phi_y \), the tip of vector \( \mathbf{E} \) may trace many different paths. **POLARIZATION** designates the state of this vector.
Polarization (2)

State 1: Linear polarization (LP)

\[ \varphi_x = \varphi_y = \varphi \]
\[ \psi = \tan^{-1} \left| \frac{E_y}{E_x} \right| \]

State 2: Right-hand circular polarization (RHCP)

\[ \varphi_y = \varphi_x - \pi / 2 \]
\[ |E_x| = |E_y| \]

State 3: Left-hand circular polarization (LHCP)

\[ \varphi_y = \varphi_x + \pi / 2 \]
\[ |E_x| = |E_y| \]
State 4: Elliptical polarization

\[ \varphi_x, \varphi_y \text{ arbitrary} \]
\[ |E_x|, |E_y| \text{ arbitrary} \]

For this arbitrary wave, define

\[ AR = \text{Axial Ratio} = \pm \frac{OA}{OB} \]

OA: Major axis; OB: Minor axis
“+” for RH polarization; “-” for LH polarization

\[ \gamma = \tan^{-1} \left( \frac{|E_y|}{|E_x|} \right) \]
\[ \delta = \varphi_y - \varphi_x \]
\[ \varepsilon = \cot^{-1}(AR) \]
\[ \tau : \text{tilt angle} \]
Poincare Sphere (1)

Jules Henry Poincare
1854-1912
French mathematician

Poincare’s sphere is a novel demonstration mechanism to show various polarization states of plane waves. Its function is similar to the Smith chart in transmission line theory. It helps to transfer between the \((\gamma, \delta)\) set and the \((\varepsilon, \tau)\) set.

- \(2\gamma\): the great-circle angle
- \(\delta\): the equator to great circle angle
- \(2\varepsilon\): latitude
- \(2\tau\): longitude
Poincare Sphere (2)

North pole: LHCP
Equator: LP
South pole: RHCP
Upper hemisphere: LH
Lower hemisphere: RH
Poincare Sphere (3)
A useful application in antenna theory

Voltage response of the antenna:

\[ V = C \cos \frac{\angle P_w P_a}{2} \]

C: Constant depending on antenna gain, distance, etc.

\( \angle P_w P_a \): Angle subtended by a great-circle arc from polarization \( P_w \) to \( P_a \).