Problem 1

Given the unity feedback system with:

\[ G_{td}(s) = K, \quad \text{and} \quad G_p = \frac{1}{s+2}. \]

Determine \( K \) so that the system has a pole at \( s = -5 \). Then find the output \( y_p(t) \) given that \( r(t) = U(t) \). Use MATLAB to verify your result.

Problem 2

Given the plant transfer function

\[ G_p(s) = \frac{1}{s+0.1}. \]

(i) Design a controller \( G_{td}(s) \) — for the open loop control system — so that \( G_p(s)G_{td}(s) \) has a pole at \( -2 \), and the output \( y_p(t) \) tracks the signal \( r(t) = r_0 U(t) \) with zero steady-state error, i.e., \( e_{ss} = 0 \).

(ii) Now suppose that the plant pole at \( s = -0.1 \) was not modeled correctly and that the correct pole is \( s = -0.2 \). Apply the control designed in part (i) and the input \( r(t) = r_0 U(t) \) to the correct plant and compute the resulting steady-state error.

(iii) Now the controller \( G_{td}(s) = \frac{2(s+0.1)}{s} \) is used with the same \( G_p(s) \) — for the unity feedback system. Verify that the closed-loop pole is \( s = -2 \). Let \( r(t) = r_0 U(t) \). Verify that the steady-state error \( e_{ss} \) is zero.

(iv) Compute the steady-state error of the closed-loop system — with plant pole at \( s = -0.2 \) — when \( r(t) = r_0 U(t) \). Compare this error to that of the correct open-loop system computed in part (ii).

Problem 3

Consider the system described by the following equations for \( t \geq 0 \):

\[
\begin{align*}
\dot{w}(t) &= az(t) + d(t), \\
\dot{z}(t) &= -aw(t) + d(t), \\
y(t) &= w(t) + z(t),
\end{align*}
\]
where $a > 0$ and $w(0) = 0 = z(0)$.
Take $u(t) = d(t)$ as the input and $y(t)$ as the corresponding output.
(i) Derive the state space description
\[
\dot{x}(t) = Ax(t) + Bu(t), \\
y(t) = Cx(t) + Du(t).
\]

(ii) Compute $e^{At}$, then solve for the state response $x(t)$, $t \geq 0$.
(iii) Calculate the Transfer Function $T(s) = \frac{Y(s)}{U(s)}$ and find its poles. Then
compute the eigenvalues of the matrix $A$. What do you conclude from what you have just found?

**Problem 4**

A non-unity FCS has
\[
G_p(s) = \frac{s + 3}{(s + 1)(s + 5)}
\]
and
\[
H(s) = K \frac{s + 4}{s + 2},
\]
where $K$ is a constant.
(i) Compute the closed loop transfer function $T_{cl}(s)$.
(ii) Is the CL system stable for all values of $K$?
(iii) Determine the steady state error $e_{ss-\text{step}}$ for all values of $K$, and for
$R(s) = \frac{1}{s}$.

**Problem 5**

The transfer function $G_p(s)$ of a plant is
\[
G_p(s) = \frac{s - 1}{s^2 - 3s + 2}.
\]
It is desire to construct a closed-loop system with the following poles: $-1$ and $-2$.
Your problem is to come up with a unity feedback system—if possible—which satisfies the given specifications.
Problem 6

Consider the system described by
\[
\dot{x}(t) = Ax(t) + Bu(t), \quad t > 0, \\
y(t) = Cx(t),
\]
where
\[
B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & -1 \end{bmatrix}.
\]
Moreover, the characteristic polynomial of $A$ is
\[p(\lambda) = \lambda^2 + \lambda,\]
and the controllability matrix $L_c$ of the pair $(A, B)$ is
\[
L_c = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.
\]

Your problem is:
(i) Show whether the pair $(C, A)$ is observable or not. If it is not, characterize the subspaces $M_o$ and $M_{uo}$.
(ii) Compute $e^{At}$, $t \geq 0$, by any method which you are most comfortable with.
(iii) Find the output $y(t)$ given that
\[
x(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{and} \quad u(t) = e^{-2t}, \quad t \geq 0.
\]