100pts, 110minutes

Your Name:

Your ID Number:

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1) (15 points) Find the Fourier transform of the periodic function \( f(t) \) below.
2) (15 points) Determine the range of values of $x$ for which the integral below will be zero.

\[ \int_{-\infty}^{\infty} \cos\left( \frac{2\pi xu}{\pi u} \right) \sin(\pi u) \, du \]
3) (30 points total)

Consider the two message signals $m_1(t)$ and $m_2(t)$ with Fourier transforms shown below.

Suppose that $m_1(t)$ and $m_2(t)$ are passed through the modulation system pictured below.
3) (Continued)

a) Provide a clearly labeled plot of $S(f)$, the Fourier transform of $s(t)$. (10 points)
3) (Continued)

Now suppose that $s(t)$ is applied to the following system:

\[
\begin{align*}
\cos(2\pi f_0 t) & \quad \rightarrow \quad y_1(t) \\
\sin(2 * 2\pi f_0 t) & \quad \rightarrow \quad y_2(t)
\end{align*}
\]

b) Provide a clearly labeled plot of $Y_1(f)$, the Fourier transform of $y_1(t)$. (10 points)
3) (Continued)

c) Provide a clearly labeled plot of $Y_d(f)$, the Fourier transform of $y_d(t)$. (10 points)
4) (20 points total)
Consider a random process that has two possible realizations as shown below.
Realization A occurs with probability \( p \). Realization B occurs with probability \( 1-p \). Both realizations are periodic.

a) Find an expression for \( E[X(t)] \) valid for the range \( 0 \leq t \leq 1 \). Note that this is NOT asking for a time average over the interval \([0,1]\). Your answer should allow evaluation, at any chosen value of \( t \) in that range, of the expected value of the process at that chosen time. (10 points)
4) (Continued)

b) Are there any values of $p$ for which this process is stationary in the mean?
   If so, specify those values. If not, explain why not. (5 points)

c) Evaluate the autocorrelation $R(t_1, t_2)$ for $t_1 = 0, t_2 = .5$ (5 points)
5) (20 points total) Consider the matched filter setup shown below:

The impulse response of $g(t)$ is as follows:

$$g(t) = \begin{cases} 
2 & 0 \leq t \leq T/2 \\
1 & T/2 < t \leq T \\
0 & \text{otherwise}
\end{cases}$$

a) Find and sketch $h(t)$, the impulse response of the matched filter, corresponding to $y(t)$. (10 points)
5) (Continued)

b) Clearly sketch and label the output, \( y(t) \), of the matched filter as a function of time.
(10 points)