Diffusion and Drift

Both Electrons and Holes contribute to conduction in a semiconductor. Both have Drift and Diffusion components. The total current density is the sum of all 4 components at a specific location:

\[
J_n(x) = q \mu_n n(x) \mathcal{E}(x) + qD_n \frac{dn(x)}{dx} \\
J_p(x) = q \mu_p p(x) \mathcal{E}(x) - qD_p \frac{dp(x)}{dx} \\
J(x) = J_n(x) + J_p(x)
\]

Assuming that \( J(x) = \text{Constant} \) is equivalent to saying that neither electrons nor holes can "pile-up" anywhere.

Note: \( \mathcal{E} \) is the electric field.
Drift and Diffusion Currents

\[ J_n(x) = q\mu_n n(x)E(x) + qD_n \frac{dn(x)}{dx} \]
\[ J_p(x) = q\mu_p p(x)E(x) - qD_p \frac{dp(x)}{dx} \]
\[ J(x) = J_n(x) + J_p(x) \]

Suppose \( n(x) \gg p(x) \). Which of the terms can be neglected?

Note that the diffusion terms can be significant for both types of carriers even if the concentration of one carrier is much larger than the concentration of the other.

Einstein Relation

By imposing the condition that a semiconductor without external connections cannot have a current flow we can show that:

\[ \frac{kT}{q} = \frac{D_n}{\mu_n} \]

This is the Einstein Relation. It gives the relationship between mobility and the diffusion coefficient (also true for \( D_p, \mu_p \)).

<table>
<thead>
<tr>
<th></th>
<th>( D_n )</th>
<th>( D_p )</th>
<th>( \mu_n )</th>
<th>( \mu_p )</th>
</tr>
</thead>
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<tr>
<td>Ge</td>
<td>100</td>
<td>50</td>
<td>3900</td>
<td>1900</td>
</tr>
<tr>
<td>Si</td>
<td>35</td>
<td>12.5</td>
<td>1350</td>
<td>480</td>
</tr>
<tr>
<td>GaAs</td>
<td>220</td>
<td>10</td>
<td>8500</td>
<td>400</td>
</tr>
</tbody>
</table>

These numbers are valid for low doping only!
PN Junction Equilibrium: Abrupt

\[
qV_0 e^{\frac{kT}{q}} = \frac{p_p}{p_n}
\]

\[
V_0 = \frac{kT}{q} \ln \left( \frac{p_p}{p_n} \right)
\]

\[p_p = N_a, n_n = N_d\]

\[p_n = \frac{n_i^2}{N_d}\]

Notice that \(V_0\) changes very little over many orders of magnitude of the doping. Using \(V_0=0.7\) is very often a good guess even if you don’t know the details of the pn junction!

Depletion width in forward and reverse bias

\[W = \frac{2\varepsilon}{q} \left( V_0 \right) \left( \frac{N_p^+ + N_n^-}{N_d^+ N_a^-} \right)\]

Under forward bias, \(V_f\), the effective contact potential is decreased by the amount of the forward bias so \(W\) is smaller.

\[W = \frac{2\varepsilon}{q} \left( V_0 - V_f \right) \left( \frac{N_p^+ + N_n^-}{N_d^+ N_a^-} \right)\]

Under reverse bias, \(-|V_r|\), the effective contact potential is increased by the amount of the reverse bias so \(W\) is larger.

For \(n+p\) or \(p+n\) junctions the term including dopant concentrations reduces to a simpler expression involving the low side doping.
Minority Carrier Injection

A positive potential produces a steady concentration of excess holes on the n side of the space charge region and excess electrons on the p side. This is minority carrier injection. The distributions of excess holes and excess electrons are determined by the diffusion equation.
Hole and Electron Flow in a n-p-n Transistor

• The particle flows in an n-p-n transistor are exactly the same as those in a p-n-p except that wherever there were holes in a pnp there are now electrons.
• The currents in an n-p-n transistor are opposite polarity.

\[ I_E = I_{En} + I_{Ep} \]
\[ \gamma = \frac{I_{En}}{I_{En} + I_{Ep}} \]
\[ I_{En} = \alpha_I \cdot I_{En} \]

Minority Carrier Injection

On the p side the minority carriers are the electrons!

\[ \frac{d^2 \Delta n}{dx^2} = \frac{\Delta n}{D_n \tau_n} = \frac{\Delta n}{L_n^2} \]

The general solution for this diff. eq. is:
\[ \Delta n(x) = A e^{x/L_n} + B e^{-x/L_n} \]

Then \( \Delta n(x) = \Delta n_p e^{-x/L_p} \) For long p region, where \( \Delta n(x = 0) \equiv \Delta n_p \)

Plug in the expression for \( \Delta n \):
\[ \Delta n(x) = n_{p0} \left( e^{qV/kT} - 1 \right) e^{-x/L_p} \]
Minority Carrier Current

With minority carriers the equilibrium concentration is so small that we can neglect the drift component. Current is proportional to the gradient of excess charge.

\[
\frac{I_n(x)}{A} = J_n(x) = qD_n \frac{d\Delta n}{dx} = -q \frac{D_n}{L_n} \Delta n_p e^{\frac{-x}{L_n}}
\]

\[
I_n(x = 0) = -qA \frac{D_n}{L_n} \Delta n_p = -qA \frac{D_n}{L_n} n_{p0} \left( \frac{e^{\frac{qV}{kT}}} - 1 \right)
\]

Similarly the minority carrier current on the p side is:

\[
I_p(x = 0) = qA \frac{D_p}{L_p} \Delta p_n = -qA \frac{D_p}{L_p} p_{n0} \left( \frac{e^{\frac{qV}{kT}}} - 1 \right)
\]

Note that both currents have the same sign!

Ideal Diode Equation

\[
I(x) \approx I_p \left( -x_n \right) + I_n \left( x_p \right) = I
\]

Plug in our expressions for \( I_p \) and \( I_n \):

\[
I = qA \left( \frac{D_p}{L_p} p_{n0} + \frac{D_n}{L_n} n_{p0} \right) \left( \frac{e^{\frac{qV}{kT}}} - 1 \right)
\]

This is the ideal diode equation. It is often written as:

\[
I = I_0 \left( e^{\frac{qV}{kT}} - 1 \right) \quad I_0 \text{ is often empirically determined.}
\]
Diffusion Equation

- The steady state diffusion equation applies to excess electrons in the base.
- We know the form of the solution...the boundary conditions are slightly different than for a long pn diode.

\[
\frac{d^2 \Delta n}{dx^2} = \frac{\Delta n}{D_n \tau_n} \equiv \Delta n \quad \Delta n(x') = A e^{x'/L_n} + B e^{-x'/L_n}
\]

\[
\Delta n(x' = 0) = A + B = \Delta n_E
\]

\[
\Delta n(x = W') = A e^{W'/L_n} + B e^{-W'/L_n} = \Delta n_C
\]

Regions of Operation of the Transistor

**Reverse Active**

**On state (saturation)**

**Cutoff**

**Forward Active**
Approximate Solution for Normal Active Mode

- When the emitter base junction is forward biased and the collector base junction is reverse biased we can obtain a simpler expression for excess electrons. This is called the normal active mode.

\[ \Delta n_E >> \Delta n_C \approx 0 \]

\[ A = \frac{\Delta n_E e^{-W'/L_n}}{e^{W'/L_n} - e^{-W'/L_n}} \quad B = \frac{\Delta n_E e^{W'/L_n}}{e^{W'/L_n} - e^{-W'/L_n}} \]

\[ \Delta n(x) = A e^{x/L_n} + B e^{-x/L_n} \]

\[ \Delta n(x) = \Delta n_E \frac{e^{W'/L_n} e^{-x'/L_n} - e^{-W'/L_n} e^{x'/L_n}}{e^{W'/L_n} - e^{-W'/L_n}} = \Delta n_E \frac{\sinh \left( \frac{W' - x'}{L_n} \right)}{\sinh \left( \frac{W'}{L_n} \right)} \]

Currents at the Three Terminals of a BJT

- Thus the three currents are given by:

\[ I_E \approx I_{En} = Aq \frac{D_n}{L_n} \left( \Delta n_C \coth(y) - \Delta n_E \coth(y) \right) \]

\[ I_C = Aq \frac{D_n}{L_n} \left( \Delta n_C \coth(y) - \Delta n_E \csch(y) \right) \]

\[ I_B = Aq \frac{D_n}{L_n} \left( \Delta n_E + \Delta n_C \right) \tanh \left( \frac{y}{2} \right) \quad y \equiv \frac{W'}{L_n} \]

These are applicable for any mode of operation of the BJT (not just normal active mode).

Notice that the emitter current is only the electron component. Can we say something about the hole component and therefore about the emitter injection efficiency, \( \gamma \), from the theory we have already developed?
Emitter Injection Efficiency Typical Value

- Why does the emitter doping need to be much higher than the base doping?

\[ \gamma = \frac{1}{1 + \frac{I_{Ep}}{I_{En}}} = \frac{1}{1 + \frac{\mu_{pE}W'}{\mu_{nB}L_{pE}N_{dE}}} \]

if \( \frac{\mu_{pE}W'N_{aB}}{\mu_{nB}L_{pE}N_{dE}} \ll 1 \) then \( \gamma \approx 1 - \frac{\mu_{pE}W'N_{aB}}{\mu_{nB}L_{pE}N_{dE}} \)

\( W' = 2 \mu m, N_a = 10^{15}, \mu_n = 1350 \)
\( L_{pE} = 10 \mu m, N_d = 10^{17}, \mu_p = 250 \)

\( \gamma \approx 1 - \frac{250}{1350} \approx 1 - 3.7 \times 10^{-4} = 0.99963 \)

Approximations of the Terminal Currents

- In the normal active mode where the emitter-base junction is forward biased and the collector-base junction is reverse biased:

\[ I_E \approx -qA \frac{D_n}{L_n} (\Delta n_E \coth(y)) \]

\[ I_C \approx -qA \frac{D_n}{L_n} (\Delta n_E \csch(y)) \]

\[ I_B \approx -qA \frac{D_n}{L_n} \Delta n_E \tanh\left(\frac{y}{2}\right) \]

\[ y \equiv \frac{W'}{L_n} \]
Base Transport Factor

- With the simplified expressions for emitter and collector current we can obtain an expression for the base transport factor.

\[
I_{Ch} = \alpha_T \cdot I_{En}
\]

\[
I_{En} \approx -qA \frac{D_n}{L_n} (\Delta n_c \coth(y))
\]

\[
I_{Cn} \approx -qA \frac{D_n}{L_n} (\Delta n_e \csch(y))
\]

\[
\frac{I_{Ch}}{I_{Ep}} = \frac{\csch(y)}{\coth(y)} = \operatorname{sech} \left( \frac{W'}{L_n} \right) = \frac{1}{\cosh \left( \frac{W'}{L_n} \right)} \approx \frac{1}{1 + \frac{1}{2} \left( \frac{W'}{L_n} \right)^2} \approx 1 - \frac{1}{2} \left( \frac{W'}{L_n} \right)^2
\]

Current Transfer Ratio and Current Gain

- The current transfer ratio was shown to be the product of the base transport factor and the emitter injection efficiency.
- The current gain is related to the transfer ratio as

\[
\beta_o = \frac{\alpha_o}{1 - \alpha_o}
\]

\[
\alpha_o = \alpha_T \gamma = \frac{1 - \frac{1}{2} \left( \frac{W'}{L_n} \right)^2}{1 + \frac{\mu_p E W' N_{ab}}{\mu_e L_p E N_{de}}}
\]

\[
W' = 2 \mu m \quad \gamma = 0.99963
\]

\[
L_n = 10 \mu m
\]

\[
\alpha_o = 0.980 \cdot 0.99963 = 0.9796
\]

\[
\beta_o = \frac{\alpha_o}{1 - \alpha_o} = \frac{0.9796}{0.0204} = 48
\]
Current Gain: Ratio of Base and Collector Current

- We should get a similar current gain from the expressions for base and collector current:

\[ I_C \approx -qA \frac{D_n}{L_n} (\Delta n_E \, \text{csch}(y)) \]

\[ W' = 2 \mu m \]

\[ L_n = 10 \mu m \]

\[ I_B \approx -qA \frac{D_n}{L_n} \Delta n_E \tanh\left(\frac{y}{2}\right) \]

\[ \beta_o = \frac{I_C}{I_B} \approx \left(\frac{\text{csch}\left(\frac{W'}{L_n}\right)}{\tanh\left(\frac{W'}{2L_n}\right)}\right)^2 \approx 2 \left(\frac{L_n}{W'}\right)^2 = 50 \]

Note that in this approximate model the ratio of collector to base current is constant with voltage and doesn’t depend on the physical size of the junction.

Note that this method for calculating beta does not take into account an emitter injection efficiency. The base current estimation assumes \( \gamma = 1 \)… It should therefore produce an overestimate of beta.

Simplified Expressions for a Good Transistor

- A good BJT has:
  - \( W'/L_n << 1 \)
  - Much higher doping in the emitter than in the base

- In a good transistor the following approximations to the various parameters are valid:

\[ \alpha_T \approx 1 - \frac{1}{2} \left(\frac{W'}{L_n}\right)^2 \quad \gamma \approx \frac{1}{1 + \frac{\mu_{pE} W' N_{aB}}{\mu_{nB} L_{pE}^n N_{dE}}} \approx 1 - \frac{\mu_{pE} W' N_{aB}}{\mu_{nB} L_{pE}^n N_{dE}} \]

\[ \alpha_o \approx 1 - \frac{1}{2} \left(\frac{W'}{L_n}\right)^2 - \frac{\mu_{pE} W' N_{aB}}{\mu_{nB} L_{pE}^n N_{dE}} \]

Generally true for \( \Delta << 1 \):

\[ \beta_o \approx 2 \left(\frac{L_n}{W'}\right)^2 \quad \text{Assuming } \gamma = 1 \quad \frac{1}{1 + \Delta} \approx 1 - \Delta \]
Ic-Vce or Common Emitter Characteristic

- Maintaining constant $I_B$ as $V_{CE}$ is reduced eventually leads to the BC junction being forward biased. $V_{BE}$ only needs to decrease slightly (note the difference in scale between left and right axes) to maintain the proper total base current.

Transistor Biasing: Load Line

A transistor used as an amplifier for small signals is biased at a quiescent operating point. The bias is sometimes obtained by introducing a load resistor between the collector and the voltage supply.
Base Transit Time

• How long does it take the average electron to diffuse across the neutral base: Base Transit Time, \( \tau_B \)?

Consider a small slice of the base \( dx' \). The average time for an electron to traverse this slice is \( dt \). We know that the current density is related to the charge density and the velocity by:

\[
J_n = -q\Delta n(x')v(x') = -q\Delta n(x')\frac{dx'}{dt}
\]

\[
dt = -\frac{q\Delta n(x')dx'}{J_n}
\]

\[
\tau_B \approx \frac{Q_B}{I_{Cn}} \approx \frac{(W')^2}{2D_n}
\]

If we add the times through all the slices from 0 to \( W' \) we will get the total transit time.

Charge Control Derivation of the Base Current

• We can also apply charge control analysis to get the base current.

Total excess charge in the base is just:

\[
Q_B = \frac{qAW'\Delta n_E}{2}
\]

The lifetime of each electron in the base is \( \tau_n \). The stored charge is replaced in this amount of time so the base current is:

\[
I_B = \frac{Q_B}{\tau_n} = \frac{qAW'\Delta n_E}{2\tau_n}
\]
Common Emitter Current Gain, $\beta$, in terms of minority carrier lifetime and base transit time

The charge in the base must be replaced...each charge is replaced (on average) every period of $\tau_n$. Charge per unit time is a current...the base current.

\[
I_B = \frac{Q_B}{\tau_n} = \frac{qAW'\Delta n_E}{2\tau_n}
\]

\[
I_C \approx I_{Cn} = AqD_n \frac{\Delta n_E}{W'} = \frac{\tau_n}{2\tau_n} \approx \frac{\tau_n}{\beta}
\]

The collector current is proportional to the slope of the excess charge.

So the current gain is equal to the ratio of minority carrier lifetime to the transit time. This number is within 2% for a beta of 50 or greater.

---

Diffusion Capacitance, Junction Capacitance

- Charge in the base is a function of applied voltage on the base emitter junction...this looks like a capacitance at the base emitter junction

\[
C_D = \frac{\left| dQ_B \right|}{dV_{BE}} = Q_B = \tau_n I_{Cn} = \alpha_o \beta I_E
\]

\[
C_D = \alpha_s \tau_B \frac{dI_E}{dV_{BE}} = \alpha_o \tau_B \frac{qI_E}{kT} = \frac{\alpha_o \tau_B}{r_E}
\]

- Charge in the base-emitter and base-collector space charge regions is a function of applied voltage on each junction.

\[
C_{EB} = \frac{dQ_{JE}}{dV_{BE}} = \frac{dQ_{JE}}{dW_{DE}} \frac{dW_{DE}}{dV_{BE}} = \frac{\varepsilon_s A}{W_{DE}}
\]
Frequency Response

• We can treat the generic case of a two port with input resistance and capacitance and an output current source and then make appropriate substitutions at the end.

\[
C_D = \frac{\alpha r_a}{r_E}
\]

The input is a current divider:

\[
v_i = \frac{r_a}{1 + j\omega C_D} = i_o \cdot \frac{r_a}{1 + j\omega C_D}
\]

The output current is just a multiple of the input voltage:

\[
i_{out} = g_m v_{in} = i_o \cdot \frac{g_m r_o}{1 + j\omega C_D r_o}
\]

\[
C_D \rightarrow C_D + C_{JE} \quad \text{If junction capacitance is significant}
\]

---

**Frequency Response**

\[
\begin{align*}
 i_{out} &= g_m v_{in} = i_o \cdot \frac{g_m r_o}{1 + j\omega C_D r_o} \\
 i_{in} &= v_{in} / C_D
\end{align*}
\]

So the current gain is a function of frequency which can be written in terms of the low frequency value (subscript o):

\[
\begin{align*}
 i_{out} &= \frac{g_m r_o}{1 + j\omega C_D r_o} \\
 i_{in} &= \frac{i_o}{1 + j\omega C_D r_o}
\end{align*}
\]

In the common base mode:

\[
\begin{align*}
 r_o &= r_E = \frac{V_T}{I_E} \\
 g_m r_o &= \frac{I_C}{V_T} = \alpha_o
\end{align*}
\]

In the common emitter mode:

\[
\begin{align*}
 r_o &= \frac{r_E}{1 - \alpha_o} = \frac{V_T}{I_E} \\
 g_m r_o &= \frac{I_C}{V_T} \left( \frac{V_T}{1 - \alpha_o} \right) = \beta_o
\end{align*}
\]

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Unity Current Gain Cutoff Frequency

- Common Emitter Mode

\[
\beta(\omega) = \frac{\beta_o}{1 + j\omega C \beta_T r_{in}}
\]

\[
f_i^* = \frac{1}{2\pi \tau_B} \left| \beta(f) \right|
\]

Also the Gain-Bandwidth Product

\( f_i \) is by far the most important parameter for characterizing high speed transistors.

Coupled Diode Model: Ebers Moll Equations

- We can think of a symmetrical npn transistor as being a superposition of two short base diodes

\[
I_E = I_{EN} + I_{EI} = I_{ES} \left( e^{q(V_{BE} - kT)/2kT} - 1 \right) - \alpha_{1I} I_{CS} \left( e^{qV_{BE} - kT}/2kT - 1 \right)
\]

\[
I_C = I_{CN} + I_{CI} = \alpha_N I_{ES} \left( e^{q(V_{BE} - kT)/2kT} - 1 \right) - I_{CS} \left( e^{qV_{BE} - kT}/2kT - 1 \right)
\]

\[\alpha_N I_{ES} = \alpha_1 I_{CS}\]
Ebers Moll Equations

- What is the common base (α) current gain in forward active mode

\[
\begin{pmatrix}
I_{ES} & -\alpha I_{CS} \\
\alpha N I_{ES} & -I_{CS}
\end{pmatrix}
\begin{pmatrix}
e^{qV_{BE}/kT} \\
e^{qV_{AC}/kT}
\end{pmatrix}
= \begin{pmatrix}
I_E \\
I_C
\end{pmatrix}
\]

\[I_{ES} e^{qV_{BE}/kT} = I_E\]
\[\alpha_N I_{ES} e^{qV_{AC}/kT} = I_C\]

\[\alpha \equiv \frac{I_C}{I_E} = \frac{\alpha_N I_{ES}}{I_{ES}} = \alpha_N\]

So the common base current gain is the ratio of \(a_{21}\) to \(a_{11}\)

Circuit Model Representing the Ebers Moll Model

- This equivalent circuit synthesizes the Ebers Moll equations.
- It makes it much easier to visualize the four terms in the equations.
Normal Active Mode: Another Equivalent Circuit

- This equivalent circuit is more appropriate for common emitter configuration
- The current gain can be obtained in terms of $\alpha_N$ as before.

\[ \beta = \frac{\alpha_N}{1 - \alpha_N} \]

Limitations

- Diode leakage and series resistance
Gummel Plot including leakage and series resistance

- At pt A shown
  \[ I_B = 1e^{-8} \text{ A} \]
  \[ V_{BE} = 0.1 \text{ V} \]
  \[ V_{BE}/I_B = R_p \]

- At pt B shown
  \[ I_B = 3e^{-2} \text{ A} \]
  \[ \Delta V_{BE} \approx 0.3 \text{ V} \]
  \[ \Delta V_{BE}/I_B = R_s \]

Voltage Limitations

- Punch Through Voltage \( V \)
  - Collector-base depletion region grows so large that there is no neutral base remaining

\[
W' = W_{BJ} - x_{pE} - x_{pC}
\]

\[
x_{pc} = \frac{2\varepsilon(N_{bc} - V_{BE})N_{BE}}{qN_{ab}(N_{ac} + N_{ab})}
\]

\[
x_{pC} = \frac{2\varepsilon(N_{bc} - V_{BE})N_{BC}}{qN_{ab}(N_{ac} + N_{ab})}
\]

\[
W' = 0 = W_{BJ} - x_{pC}(V_{BE} = V_{BC}) = x_{pC}(V_{BC} = V_{pt})
\]

Usually one of the depletion regions is much larger than the other and dominates this expression. For devices in forward active mode you are often seeking:

\[ V_{BC} \approx V_{pt}, x_{pC}(V_{BC} = V_{pt}) \approx W_{BJ} \]
**Voltage Limitations**

- **Example of Punch Through Calculation for a specific base doping**

\[ V_{BE} = 0.5V \]

\[ N_{BE} = 1 \times 10^{19} / cm^3 \]

\[ N_{eff} = 1 \times 10^{17} / cm^3 \]

- Note that low collector doping gives very high punch through voltage...Why?
  Ans. Majority of depletion is on the collector side (not the base side) added voltage mostly depletes collector

- At collector doping above base doping additional doping does not degrade punch through any further...Why?
  Ans. Now all depletion is on base side added voltage depletes only the base

- For very thin metallurgical junctions punch through voltage is very low...Why?
  Ans. There is some depletion on the emitter side, it plays a much larger role as the base is thinned.