EE121B
Principles of Semiconductor Device Design

Lecture 5

BJT RF and Transient Behavior
Course Instructor: Marko Sokolich

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University of California Los Angeles

UCLA

BJT Models, Transistor Switching, Limitations

- Frequency Response
- Cutoff Frequency
- Junction Capacitances
- Break
- Switching Transistor
Last Time: Base Transit Time

- In this lecture we will learn that Base transit time is a key to understanding the RF behavior of a transistor.

We saw that by slicing up the base into small slivers we could obtain an expression for the average time it takes for an electron to diffuse across the base.

\[ \tau_B = \frac{Q_B}{I_n} \]

In the usual case of very small recombination in the base, \( \alpha_r \approx 1 \)

\[ Q_B = -\frac{qAW_n^2}{2} \]

\[ I_s \approx I_{Cn} = AqD_n \frac{d\Delta n}{dx'} \bigg|_{x'=0} \]

\[ I_{Cn} = AqD_n \frac{\Delta n_n}{W'} \]

\[ \tau_B = \frac{Q_B}{I_{Cn}} = \frac{(W')^2}{2D_n} \]

Notice that base transit time is proportional to the square of the base width, \( W' \).

Last Time: Transistor Biasing & Load Line

A transistor used as an amplifier for small signals is biased at a quiescent operating point.

The bias can be achieved by introducing a load resistor between the collector and the voltage supply.

How does changing the bias affect the small signal model?

\[ r_E = \frac{kT}{qI_E} = \frac{V_I}{I_E} \]

\[ g_m = \frac{qI_C}{kT} = \frac{I_C}{V_I} \]
Last Time: Diffusion Capacitance

- Modulating the voltage on the emitter base junction modulates the excess charge in the neutral base...dQ/dV is a capacitance.

\[
C_D = \frac{dQ_B}{dV_{BE}} \quad Q_B = \tau_B I_C \quad \alpha \tau_B I_E
\]

\[
C_D = \alpha \tau_B \frac{dI_E}{dV_{BE}} = \alpha \tau_B \frac{qI_E}{kT} = \frac{\alpha \tau_B}{r_E} \quad C_D = \frac{\alpha \tau_B}{r_E}
\]

Last time:
Upgrading the Common Base Small Signal Model

- We can now add the diffusion capacitance to the small signal model
- This model for the intrinsic transistor is now valid for any frequency
- The intrinsic transistor is essentially defined by the neutral base and contains everything we need for transistor action.

As we will see later, there are many parasitic elements that will degrade this transistor... in effect, this is the best that we can do in a BJT.
Last Time:
Upgrading the Common Emitter Small Signal Model

• The CE model incorporates the diffusion capacitance in the same way between emitter and base terminal
• The input current is now the base current and sees an input resistance that is larger by a factor of \(1/(1-\alpha_0)\).

\[
\frac{I_b}{V_L} = \frac{1}{r_E} \quad \alpha \quad V_{in} \quad i_b \quad i_e \quad + \quad + \quad v_{out} \quad R_L \quad - \quad -
\]

Frequency Response

• We can treat the generic case of a two port with input resistance and capacitance and an output current source and then make appropriate substitutions at the end.

\[
C_D = \frac{\alpha_0 r_B}{r_E}
\]

The input is a current divider:

\[
v_{in} = i_{in} \left( \frac{1}{r_C} \right)
\]

The output current is just a multiple of the input voltage:

\[
i_{out} = g_m v_{in} = i_{in} \left( \frac{g_m r_m}{1 + j0C\omega r_m} \right)
\]
Frequency Response

So the current gain is a function of frequency which can be written in terms of the low frequency value (subscript o):

\[ \frac{i_{out}}{i_{in}} = \frac{g_m r_m}{1 + j \omega C_D r_m} = \left( \frac{i_{out}}{i_{in}} \right)_o \]

In the common base mode:

\[ r_m = r_e = \frac{V_T}{I_T} \]

\[ g_m r_m = \frac{I_C}{V_T} \frac{V_T}{I_T} = \alpha_o \]

In the common emitter mode:

\[ r_m = \frac{r_e}{1 - \alpha_o} = \frac{V_T}{(1 - \alpha_o)I_T} \]

\[ g_m r_m = \frac{I_C}{V_T} \frac{V_T}{(1 - \alpha_o)I_T} = \beta_o \]

Cutoff Frequency

- Common Base Mode

\[ \alpha(\omega) = \frac{\alpha_o}{1 + j \omega C_D r_m} \]

\[ \omega_c^* = \frac{1}{C_D r_m} \]

\[ \alpha(\omega) = \frac{\alpha_o}{1 + j \omega C_D r_m} = \frac{\alpha_o}{1 + j \frac{\omega}{\omega_c^*}} \]

Alpha cutoff Frequency

\[ f_c^* = \frac{\omega_c}{2\pi} = \frac{1}{2\pi \alpha_o \tau_B} \]
Cutoff Frequency

• Common Emitter Mode

\[ r_m = r_e \frac{V_r}{1 - \alpha_e} \]
\[ g_m' = \frac{I_C}{V_r} \left( \frac{V_r}{1 - \alpha_e} \right) = \beta_c \]
\[ \beta(\omega) = \frac{\beta_c}{1 + j \omega C_{D} r_{in}} \]
\[ \omega_{c} = \frac{(1 - \alpha_o)}{C_o r_E} \]
\[ \beta(\omega) = \frac{\beta_o}{1 + j \omega C_o r_k} = \frac{1}{1 + j \frac{\omega}{\omega_{c}}} \]

Beta cutoff Frequency
\[ f_{c} = \frac{\omega_{c}}{2\pi} = \frac{1}{2\pi \beta_o \tau_B} \]

Unity Current Gain Cutoff Frequency

• Common Emitter Mode

\[ r_m = r_e \frac{V_r}{1 - \alpha_e} \]
\[ g_m' = \frac{I_C}{V_r} \left( \frac{V_r}{1 - \alpha_e} \right) = \beta_c \]
\[ \beta(\omega) = \frac{\beta_o}{1 + j \omega C_{D} r_{in}} \]
\[ f_{c} = \frac{1}{2\pi \tau_B} \]

Also the Gain-Bandwidth Product
\[ f_c = 100 \text{ GHz} \]

\[ f_c \text{ is by far the most important parameter for characterizing high speed transistors} \]
Junction Capacitances

• You may recall from your study of pn junctions that there is a capacitance associated with the space charge region of the junction.

The space charge region has equal and opposite charges on either side.

In a uniformly doped semiconductor the charge on each side is proportional to the depletion layer width on that side.

The depletion layer width is modulated by the potential across the junction.

So the voltage is modulating the charge...a capacitance.

\[ C_{JEB} = \left| \frac{dQ_{JE}}{dV_{BE}} \right| \quad C_{JCB} = \left| \frac{dQ_{JC}}{dV_{CB}} \right| \]

Junction Capacitance: Calculating from Depletion Width

• We have all the necessary information to calculate \( C_{JEB} \)
• The notes do not do this analysis...but it is very important

Recall the relationships between \( n \) and \( p \) side depletion region widths and the total depletion width in a pn junction

\[ x_{n0} = \frac{WN_n^-}{(N_p^+ + N_n^-)} \]
\[ x_{p0} = \frac{WN_p^+}{(N_p^+ + N_n^-)} \]

\[ W_e = \sqrt{\frac{2e(V_n - V_{BE})}{e}} \left( \frac{N_p^+ + N_n^-}{N_p^+ N_n^-} \right) \]

Total + charge on n-side is just volume times charge/volume

\[ Q_n = A x_{n0} N_n^- = \frac{W_e N_p^+ N_n^-}{(N_p^+ + N_n^-)} \]

\[ C_{JEB} = \left| \frac{dQ_n}{dV_{BE}} \right| = \left| A \frac{N_p^+ N_n^-}{(N_p^+ + N_n^-)} \frac{dW_e}{dV_{BE}} \right| \]

We get the same formula as for two parallel plates separated by a distance \( W_e \). The correct \( W_e \) is the \( W_e \) evaluated at the bias point \( W_e(V_{BE}) \).
Junction Capacitance:
Modifications to Common Base Small-Signal Model

- In the common base model the two junction capacitances are completely analogous because both input and output are referred to the base terminal.

\[ C_{JEB} = \frac{eA}{W_e} \quad C_D = \frac{\alpha \tau_B}{r_E} \quad I_C = \frac{V_T}{r_E} \quad C_{JCB} = \frac{eA}{W_c} \]

Cutoff Frequency: Including \( C_{JEB} \)

- Common Base Mode

\[ \alpha(\omega) = \frac{\alpha_o}{1 + j \omega (C_D + C_{JEB}) r_{in}} \]

\[ \omega_{ca} = \frac{1}{(C_D + C_{JEB}) r_E} \]

\[ \alpha(\omega) = \frac{\alpha_o}{1 + j \omega (C_D + C_{JEB}) r_E} = \frac{\alpha_o}{1 + j \omega \omega_{ca}} \]

\[ \alpha \text{ cutoff frequency} \quad f_{ca} = \frac{\omega_{ca}}{2\pi} = \frac{C_D}{C_D + C_{JEB}} \frac{1}{2\pi \alpha_o \tau_B} \]

The cutoff frequency is scaled by this fraction...<1
Common Emitter: Miller Effect Capacitance

- Now the capacitance of the collector base junction appears across the input and output circuits

\[ C_{JEB} = \frac{\varepsilon A}{W_e(V_{BE})} \]
\[ C_o = \frac{\alpha_e r_b}{r_e} \]
\[ I_e = \frac{1}{r_e} \]
\[ C_{JCB} = \frac{\varepsilon A}{W_e(V_{BC})} \]

We can convert this to an input and an output shunt capacitance by using Miller’s Theorem…but that is something for EE115

Just in case you are curious

- The shunt capacitance at the input is the junction capacitance multiplied by the voltage gain of the circuit
- The shunt capacitance at the output is very nearly equal to the junction capacitance if \( g_m R_L \) is large

\[ C_{JEB} \]
\[ C_{JCB} \]
\[ g_m R_L C_{JCB} \]
\[ g_m R_L >> 1 \]
Unity Current Gain Cutoff Frequency

\[ r_n = \frac{r_e}{1 - \alpha_o} = \frac{V_T}{(1 - \alpha_o)I_E} \]

\[ g_m r_n = \frac{l_c}{V_T} \left( \frac{V_T}{(1 - \alpha_o)I_E} \right) = \beta_o \]

\[
\beta(\omega) = \frac{\beta_o}{1 + j\omega C_{in} r_n}
\]

\[ C_{in} = C_D + C_{JE} + g_m R_L C_{JBC} \]

\[ C_{in} = C_D + C_{JE} + \frac{\beta_o}{R_L} C_{JBC} \]

\[ C_{in} r_n = \beta_o \tau_B + \frac{r_E}{1 - \alpha_o} C_{JEB} + \beta_o R_L C_{JBC} \]

\[ C_{in} r_n = \left( \frac{r_E}{1 - \alpha_o} \right) \left( \frac{\alpha_s \tau_B}{r_E} \right) + \left( \frac{r_E}{1 - \alpha_o} \right) C_{JEB} + \beta_o R_L C_{JBC} \]

\[ C_D = \frac{\alpha_s \tau_B}{r_E} \]

\[ C_{JEB} = \frac{\varepsilon A}{W_E} \]

\[ C_{JBC} = \frac{\varepsilon A}{W_C} \]
Switching Behavior

Regions of Operation of the Transistor

Reverse Active

On state (saturation)

Cutoff

Forward Active
Switching

- A BJT can be operated as a switch
- In the “On” state current passes easily from emitter to collector.
- In the “Off” state there is negligible current flow.
- The “On” and “Off” states are controlled by the biasing of the base terminal.

\[ I_E \quad I_C \]

“On” state  “Off” state

| High current, low voltage | High voltage, no current |

Switching: Qualitative Behavior

- Switching circuit for common-emitter BJT.
- With \( e_s \) low the BE junction is reverse biased and no current flows in the collector. The voltage drop across the transistor is 40V
- With \( e_s \) high the BE junction is forward biased and current flows in the collector. The voltage drop across the transistor is very small (~\( V_{BE} \))

\[ i_B = 0.10 \text{ mA} \]

<table>
<thead>
<tr>
<th>( i_C (\text{mA}) )</th>
<th>( -V_{CE} (V) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>-0.02</td>
</tr>
<tr>
<td>0.06</td>
<td>-0.04</td>
</tr>
<tr>
<td>0.04</td>
<td>-0.06</td>
</tr>
<tr>
<td>0.02</td>
<td>-0.08</td>
</tr>
</tbody>
</table>

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Switching: Cutoff

- In cutoff both pn junctions in the BJT are reverse biased.
- All of the charge is sucked out of the base so that the excess charge is actually negative.

\[ \Delta n_C = -n_p \]

Using the charge storage approach we can estimate the base current to be:

\[ B_{pb} = qAW_b\Delta n_E = \frac{-qAW_bn_p}{\tau_p} = I_B \]

Switching: Saturation

- The saturation regime begins when the reverse bias across the collector junction is reduced to zero.

In saturation there is very little voltage dropped from emitter to collector so most of the voltage drop in the circuit is across the resistor.

The base current increases and therefore the stored charge increases.
Switching: Excess Holes in the Base

- As the BJT progresses through the switching cycle excess electrons build up in the base.

\[ \Delta n(t) \]

- \( t_0 \) – Cutoff
- \( t_1 \) – Normal Active Region
- \( t_s \) – Beginning of Saturation
- \( t_2 \) – Final saturated state

Switching Transients

- As soon as \( V_{CB} \) reaches zero the collector current is at a maximum.
- The stored charge in the base, however, continues to increase.
- When the base current is turned off, the collector current does not immediately drop.

\[ i_C(t) \sim \frac{E_{CE}}{R_L} \]
Charge Control Derivation of the Base Current

- We can also apply charge control analysis to get the base current.

\[ i_B \]

\[ \tau_n \]

The lifetime of each electron in the base is \( \tau_n \). The stored charge is replaced in this amount of time so the base current in steady state is:

\[ I_B = \frac{Q_B}{\tau_n} = \frac{qAW'\Delta n_E}{2\tau_n} \]

In the transient case this becomes:

\[ I_B = \frac{dQ_B}{dt} + \frac{Q_R}{\tau_n} \]

Charge control equation

Next Time: Models and Limitations

- Ebers Moll Model
- Gummel Poon Model