HW # 6 Solution

1) d = Signal

When d is HIGH

\[ R_{up} = \frac{R_{up}'}{2 \mu m} = \frac{3 \text{ k} \Omega \cdot \text{um}}{2 \mu m} = 1.5 \text{ k} \Omega \]

\[ R_{down} = \frac{R_{down}'}{2 \mu m} = \frac{2.4 \text{ k} \Omega \cdot \text{um}}{2 \mu m} = 1.2 \text{ k} \Omega \]

\[ C_1 = R_{up} C_{at} \]

\[ = \left( 666.67 \text{ um} \right) \left( \frac{12 \mu m}{2 \mu m} \right) \]

\[ = 16 \text{ ps} \]

\[ C_2 = R_{down} C_{tot} \]

\[ = \left( 600 \text{ um} \right) \left( \frac{22 \mu m \times 2.5 \mu m}{2 \mu m} \right) \]

\[ = 26.4 \text{ ps} \]

\[ C_3 = R_{up} C_{tot} \]

\[ = \left( 300 \text{ um} \right) \left( 14 \mu m \times 2.5 \mu m \right) \]

\[ = 8.4 \text{ ps} \]
\[ T_{PHL} = 0.69 (T_1 + T_2 + T_3) \]
\[ = 0.69 \left( 16ps + 26.4ps + 8.4ps \right) = 35.052ps \]

When \( d \_s \) is Low

\[ R_{pdw} = \frac{R_{pdw}'}{2nm} = \frac{8k\Omega \cdot \mu m}{2nm} = 3 \Omega \]
\[ R_{wdw} = \frac{R_{wdw}'}{2nm} = \frac{1.2k\Omega \cdot \mu m}{2nm} = 1.8 \Omega \]
\[ R_{L} = R_{pdw} + R_{wdw} = 500 \Omega \]

\[ T_1 = \text{Reg Ctr1} \]
\[ = (500 \Omega) \left( 12 \mu m \times \frac{2 \text{ff}}{\mu m} \right) \]
\[ = 12 \text{ps} \]

\[ T_2 = \text{Reg Ctr2} \]
\[ = (600 \Omega) \left( 22 \mu m \times \frac{2 \text{ff}}{\mu m} \right) \]
\[ = 26.4 \text{ps} \]

\[ T_3 = \text{Regw Ctr} \]
\[ = (100 \Omega) \left( 14 \mu m \times \frac{2 \text{ff}}{\mu m} \right) \]
\[ = 8.4 \text{ps} \]

\[ T_{PHL} = 0.69 \left( T_1 + T_2 + T_3 \right) = 0.69 \left( 16.8 \text{ps} \right) = 32.292 \text{ps} \]
When \( \text{ck} = \text{Low} \), the second pass transistor is off and whatever that was stored in the register is stored in the 2nd feed back.
2)

a) \( T_{cyclem} > 2 + 9 + 1 \)

\[ T_{cyclem} > 12 \]

b) \( S_{pd} > T_{hold} - S_{ca} + |t_{skew}| \)

Case 1: between \( ck_1 \) and \( ck_2 \)

Case 2: between \( ck_1 \) and \( ck_3 \)

Both have a minimum propagation delay of 1 in logic 1.

\[ 1 > 1 - 0.5 + |t_{skew}| \]

\[ 0.5 > |t_{skew}| \]

\[ T_{cyclem} > 12 - 0.5 = 11.5 \] for \( t_{skew} > 0 \)

\[ T_{cyclem} > 12 + 0.5 = 12.5 \] for \( t_{skew} < 0 \)

Choose \[ T_{cyclem} > 12.5 \]

c) 

For logic 1

\[ T_{hold} + |lock_{01}| < S_{ca} + S_{pd} = 4 \]

\[ |lock_{01}| < 0.5 \]

For logic 2

\[ T_{hold} + |lock_{21}| < S_{ca} + S_{pd} = 4 \]

\[ |lock_{21}| < 3.5 \]

In order not to violate the hold time constraint, choose

\[ |lock_{21}| < 0.5 \]
if \( \text{ack}_{21} = 0.5 \)

for logic 1 \( T_{cycle\min} > 2 + 6 + 1 - 0.5 = 8.5 \)

for logic 2 \( T_{cycle\min} > 2 + 8 + 1 - 0.5 = 10.5 \)

So if \( \text{ack}_{21} < 0 \), then \( T_{cycle} = 10.5 \)

But if \( \text{ack}_{21} = -0.5 \)

for logic 1 \( T_{cycle\min} > 2 + 6 + 1 + 0.5 = 9.5 \)

for logic 2 \( T_{cycle\min} > 2 + 8 + 1 - 0.5 = 10.5 \)

So if \( \text{ack}_{21} < 0 \), then \( T_{cycle} = 10.5 \)
So from the previous page, $T_{cycle\min}$ can be reduced down to 10.5 if $\Delta CK_{21} = -0.5$. However, that cycle time will not meet the constraint for the propagation delay of 9 between top register $CK1$ and top register $CK3$.

$\Delta CK_{21} = \text{opd} = 9 \quad t_c = 1$

$|\Delta CK_{31}| = 0.5$ because the min delay between $CK1 \& CK3$ is 1.

$10.5 > \frac{2 + 9 + 1 - 0.5}{11.5}$

This condition doesn't meet.

This means that you need to raise the previous calculated $T_{cycle\min}$ to 11.5 in order to satisfy the above condition.

$T_{cycle\min} = 11.5$

CK3 needs to move back with respect to CK1 to give more time in the logic.

If people assume that CK3 is fixed with respect to CK1, then $T_{cycle\min} = 12$.

If people don't consider the min delay from bottom left register to top register (controlled by CK3), then it is possible to achieve $T_{cycle\min} = 10.5$ because $\Delta CK_{31} = +1.5$. 

\[ T_{cycle\min} = 10.5 \]
In this case, node A will switch from high to low some delay after "in" switches from low to high. Because of this, the two inputs to the NOR gate will be high together briefly and then node A will be low while "in" is high. "in" at node A will never be zero together and so "out" will not switch, and thus no power dissipation contributed by the NOR gate.

\[ C_1 = 6C_0' + 6C_0' = C_2 = 24 \text{ fF} \]
\[ C_3 = 6C_0' + 10C_0' = 32 \text{ fF} \]

\[ E_{dyne} = \frac{1}{2} C_1 V_{dd}^2 + \frac{1}{2} C_2 V_{dd}^2 + \frac{1}{2} C_3 V_{dd}^2 \]
\[ = \frac{1}{2} (24 \text{ fF})(2.5)^2 + \frac{1}{2} (24 \text{ fF})(2.5)^2 + \frac{1}{2} (32 \text{ fF})(2.5)^2 \]
\[ = \boxed{0.25 \text{ pJ}} \]
In this case, "Out" will switch because "in" and node A will be zero for a brief period of time.

\[
E_{\text{dyne}} = \frac{1}{2} C_1 V_{\text{dd}}^2 + \frac{1}{2} C_2 V_{\text{dd}}^2 + \frac{1}{2} C_3 V_{\text{dd}}^2 + (C_{\text{load}} + 12 \times 2) V_{\text{dd}}^2
\]

\[
= 0.25 \text{ pJ} + (20 \text{ pF} + 24 \text{ pF}) \times 2.5^2
\]

\[
= 0.525 \text{ pJ}
\]

C) you need to consider both transitions of the input.

So, \[P_{\text{dyne}} = \left[ E_{\text{dyne}} (\text{in 1\rightarrow 0}) + E_{\text{dyne}} (\text{in 0\rightarrow 1}) \right] \times f\]

\[
= \left[ 0.525 \text{ pJ} + 0.25 \text{ pJ} \right] \times 1 \times 10^9 \times \frac{1}{3}
\]

\[
= 0.775 \text{ mW}
\]
4)

\[ a \quad b \quad c \quad e \quad f \]

\[ P(a=1) = \frac{1}{2} \]
\[ P(b=1) = \frac{1}{4} \]
\[ P(c=1) = \frac{1}{3} \]

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<tr>
<th>b</th>
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\[ P(z=1) = P(b=1) \times P(c=1) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12} \]
\[ P(z=0) = 1 - \frac{1}{12} = \frac{11}{12} \]

Probability that \( z \) is one is

\[ P(z=1) = P(b=1) \times P(c=1) = \frac{1}{4} \times \frac{1}{3} = \frac{1}{12} \]
\[ P(z=0) = 1 - \frac{1}{12} = \frac{11}{12} \]

Probability that \( f \) is one is when both inputs are zero.

\[ P(f=1) = P(a=0) \times P(z=0) \]
\[ = \frac{1}{2} \times \frac{11}{12} = \frac{11}{24} \]
\[ P(f=0) = 1 - \frac{11}{24} = \frac{13}{24} \]

\[ table \]

\[ \alpha_{001} = P(f=0) \times P(f=1) \]
\[ = \frac{13}{24} \times \frac{11}{24} = \frac{143}{576} \]
a) 

\[ I_{\text{leak}} = 50 \text{pA/} \mu \text{m} \quad \text{w/} \quad V_{\text{th}} = 0.2 \text{V} \quad \text{Temp} = 300 \text{K}, \quad V_{GS} = 0 \]

\[ n = 1.3, \quad \tau = 0.2, \quad \Phi = 0.6 \text{V} \]

\[ V_{GS} = 0 \]

\[ \frac{I_{\text{leak}} (400K)}{I_{\text{leak}} (700K)} = \frac{e^{\frac{q(0-V_T)}{nk(400)}}}{e^{\frac{q(0-V_T)}{nk(700)}}} \]

\[ I_{\text{leak}} (400K) = 50 \text{pA/} \mu \text{m} \times e^{-\frac{q(0-V_T)}{nk(400)}(\frac{1}{400} - \frac{1}{300})} \]

\[ = 50 \text{pA/} \mu \text{m} \times e^{-\frac{(1.6 \times 10^{-19})(0.3)}{(1.3)(1.32 \times 10^{-23})}(\frac{1}{400} - \frac{1}{300})} \]

\[ = 464.8 \text{pA/} \mu \text{m} \]

b) 

\[ V_{SS} = 2 \text{V} \]

\[ V_T = V_{To} + \delta \left( \sqrt{V_{SS} + 2\Phi} - \sqrt{2\Phi} \right) \]

\[ = 0.3 + 0.2 \left( \sqrt{2 \text{V} + 0.6} - \sqrt{0.6} \right) \]

\[ V_T = 0.418 \text{V} \]

\[ \frac{I_{\text{leak}} \text{ (new } V_T)}{I_{\text{leak}} \text{ ( } V_T=0.3)} = \frac{e^{\frac{q(0-V_T)}{nk(300)}}}{e^{\frac{q(0-V_{\text{new}})}{nk(300)}}} \]

\[ I_{\text{leak}} \text{ (new } V_T) \]

\[ = \left[ e^{\frac{q}{nk(300)}(-V_{\text{new}} + V_{\text{old}})} \right] \times I_{\text{leak}} (V_T=0.3) \]

\[ = \left[ e^{\frac{(1.6 \times 10^{-19})}{(1.3)(1.32 \times 10^{-23})}(0.38 + 0.3)} \right] \times 50 \text{pA/} \mu \text{m} \]

\[ = 0.339 \text{pA/} \mu \text{m} \]

With increasing \( V_T \), the leakage current ↓
6) a) for short channel

\((W, L, \text{ and } t_{on})\) scale by \(\frac{1}{s}\)

\(V_{dd}, V_{t} \text{ don't scale}\)

\(\Rightarrow\) Fixed-Voltage scaling

Delay before is proportional to \(RC\)

\[C_{before} \sim C_{ox}WL\]

\[C_{ox} \rightarrow 5C_{ox} \text{ after scaling}\]

\[W \rightarrow \frac{W}{5}\]

\[L \rightarrow \frac{L}{5}\]

\[C_{after} \sim \frac{C_{before}}{5}\]

\[R_{on before} = \frac{V}{I_{sat}}\]

\[I_{sat} \sim C_{ox}WV\]

\[C_{ox} \rightarrow 5C_{ox} \text{ after scaling}\]

\[W \rightarrow \frac{W}{5}\]

\[I_{sat} \text{ remains unchanged after scaling}\]

\(\Rightarrow R_{on after} = \frac{R_{on before}}{5}\)

Therefore

\[
\text{Delay}_{after} = \frac{\text{Delay}_{before}}{5}
\]

Power = \(V \times I_{sat}\)

Before scaling, both \(V\) and \(I_{sat}\) remain unchanged.

So

\[
\text{Power}_{after} = \frac{\text{Power}_{before}}{5}
\]
b) $L \downarrow$ by a factor of $S$

- $C_{ox}$ stays the same
- $W$ stays the same
- $L \Rightarrow \frac{L}{S}$

- Current $\propto \frac{Current\ before}{S}$
- $V_{DD}$ stays the same

So $Delay_{after} = \frac{Delay_{before}}{S}$

Power_{after} = Power_{before}$ because $V + I$ set unchanged

c) Similar to the equation on page 226 of the book

$$\text{EDP}_{\text{for pullup}} = \frac{C^2 V_{DD}^3}{2 K_1 (V_{DD} + V_T + V_{Ssat})^2}$$

$$K_1 = \frac{V_{Ssat} V_T}{C_{ox}}$$

$$\frac{d}{dV_{DD}} \text{EDP}_{\text{for pullup}} = \frac{3 C^2 V_{DD}^2}{2 K_1 (V_{DD} + V_T + V_{Ssat})^2} - \frac{C^2 V_{DD}^3}{2 K_1 (V_{DD} + V_T + V_{Ssat})^2}$$

$$3 \frac{C^2 V_{DD}^2}{2 K_1 (V_{DD} + V_T + V_{Ssat})} = \beta V_{DD}^2$$

$$V_{DD} = \frac{3}{2} (V_T + V_{Ssat})$$

$$V_{Dsat} = -3 \beta V_{DD}$$

EDP_{for pull-down} is derived the same way.

$$V_{DD} = \frac{3}{2} (V_{in} + V_{Ssat})$$

$$V_{DDopt} = \frac{3}{2} \left( -V_T - \frac{V_{Ssat} + V_T + V_{Ssat}}{2} \right)$$

$$= \frac{3}{2} \left( 10.5 + 0.5(0.5) + 0.5 + 4(0.2) \right) = 2.25 \text{ V}$$
6) d) \( V_{THL} \) increases

Fast PMOS means there is more current pulling high

\( V_{THL} \) shifts to the right when PMOS is stronger than NMOS

For part a), if we talk about long channel, so not as easy to be in saturation.

\[
\text{then } \quad I_{sat} \propto \frac{Cox W}{L} V \\
\text{so } \quad R_{on\ before} = \frac{V}{I_{sat}} \\
Cox \rightarrow s \cdot Cox \\
W \rightarrow \frac{W}{s} \\
L \rightarrow \frac{L}{s} \\
\Rightarrow \quad I_{sat\ after} \propto s \cdot I_{sat\ before} \\
R_{on\ after} = \frac{1}{s} \cdot R_{on\ before} \\
\text{Therefore } \quad \text{Delay after} = \frac{Cox \times R_{on\ before}}{s^2} \\
\text{Power after} = \frac{5 \cdot \text{Power before}}{s}
\]

part b) when \( L \) is by \( s \) \( W \times \) Cox fixed

\[
\text{Cox \& } s \text{ \& } L \\
\text{Delay after} = \frac{Cox \times R_{on\ before}}{s} \\
\text{Power after} = \frac{5 \cdot \text{Power before}}{s}
\]