Problem 1: 20
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Problem 5: 20

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Total : 100
Problem 1 (20 Points)

Assuming that the diodes in the circuit shown below are ideal, sketch the output for the input shown. Label the most positive and most negative output levels.

(a) RC >> T
(b) RC = T/2

(a) First, find the output at $V'_\text{out}$. When $RC >> T$, $\tau_1 = 2RC >> T$, $\tau_2 = 3RC >> T$

$$V'_1 = 8\,V$$

$$-V'_2 = -12\,V$$

$$\begin{cases} V'_1 + V'_2 = 20 \\ V'_1 = \frac{V'_2}{\tau_1} = \frac{3V'_1}{\tau_2} \Rightarrow \begin{cases} V'_1 + V'_2 = 20 \\ 3V'_1 = 2V'_2 \end{cases} \Rightarrow \begin{cases} V'_1 = 8 \\ V'_2 = 12 \end{cases} \end{cases}$$

by voltage divider $\Rightarrow \begin{cases} V_1 = \frac{1}{2}V'_1 = 4 \\ V_2 = \frac{2}{3}V'_2 = 8 \end{cases}$

(b) First, find the output at $V'_\text{out}$. When $RC = T/2$, $\tau_1 = 2RC = T$, $\tau_2 = 3RC = 1.5T$

$$V'_1 = 12.00\,V$$

$$-V'_2 = -15.39\,V$$

$$\begin{cases} V'_1 e^{-\frac{T}{\tau_1}} + V'_2 = 20 \\ V'_1 e^{-\frac{T}{\tau_1}} + V'_2 e^{-\frac{1.5T}{\tau_2}} = 20 \end{cases} \Rightarrow \begin{cases} V'_1 e^{-\frac{T}{\tau_1}} + V'_2 = 20 \\ V'_1 + 0.5134V'_2 = 20 \end{cases} \Rightarrow \begin{cases} V'_1 = 0.3679V'_1 + V'_2 = 20 \\ V'_1 = 12.00 \end{cases} \Rightarrow \begin{cases} V'_1 = 12.00 \\ V'_2 = 15.59 \end{cases}$$

By voltage divider $\Rightarrow \begin{cases} V_1 = \frac{1}{2}V'_1 = 6.00 \\ V_2 = \frac{2}{3}V'_2 = 10.39 \end{cases}$
**Problem 2 (20 Points)**

Sketch and label the transfer characteristic of the circuit shown for $-6V \leq v_i \leq +6V$. Assume that conducting diodes can be represented by the constant-voltage-drop model ($V_D = 0.7 \text{ V}$). What are the slopes of the characteristic?

(a) When $-6V \leq v_i \leq -2.7V$, $D_2$ is on and $D_1$ is off.

\[
v_o = v_i + (-2 - v_i - V_D) \frac{3}{3 + 2} = 0.4(v_i + 6) - 4.02
\]

\[\therefore \text{slope} = 0.4\]

(b) When $-2.7V \leq v_i \leq 1.7V$, both $D_1$ and $D_2$ are off.

\[v_o = v_i\]

\[\therefore \text{slope} = 1\]

(c) When $1.7V \leq v_i \leq 6V$, $D_1$ is on and $D_2$ is off.

\[
v_o = 1 + (v_i - 1 - V_D) \frac{1}{3 + 1} + V_D = 0.25(v_i - 1.7) + 1.7
\]

\[\therefore \text{slope} = 0.25\]
Problem 3 (20 Points)

For the circuit shown, assume that the transistor has a $\beta = 100$ and $V_{CE_{\text{sat}}} = 0.3$ V.

(a) Select suitable values for $R_C$ and $R_B$ such that the transistor is biased at the edge of saturation at a collector current of 3 mA.

(b) Select suitable values for $R_C$ and $R_B$ such that the transistor is biased at a collector current of 1.5 mA, and so that $V_C$ is 0 V.

(c) A sinusoidal signal source is now inserted in the same circuit as shown in Figure (2). Assume resistor values to be those as designed in part (b). What is the maximum possible unclipped amplitude a sinusoid can have at the output?

\begin{align*}
\text{(a)} & \quad \text{At the edge of saturation, } V_{CE} = V_{CE_{\text{sat}}} = 0.3 \text{ V.} \\
I_c = 3mA \Rightarrow I_B = 0.03mA, I_E = 3.03mA \\
V_c = V_{CE} - I_cR_c - V_D = 9.3 - 3R_c, V_E = -V_{EE} + I_E R_E = -10 + 3.03 \times (3R_c / 4) \\
\Rightarrow V_{CE} = V_c - V_E = 19.3 - 5.2725R_c = 0.3 \Rightarrow R_c = 3.60k\Omega \\
\Rightarrow V_E = -1.81V \Rightarrow V_B = V_E + 0.7 = -1.11V \Rightarrow R_B = (-V_B) / I_B = 37.03k\Omega \\
\text{(b)} & \quad I_c = 1.5mA \Rightarrow I_B = 0.015mA, I_E = 1.515mA \\
V_c = V_{CC} - I_cR_c - V_D = 9.3 - 1.5R_c = 0 \Rightarrow R_c = 6.2k\Omega \\
\Rightarrow V_E = -2.955V \Rightarrow V_B = V_E + 0.7 = -2.255V \Rightarrow R_B = (-V_B) / I_B = 150k\Omega \\
\text{(b)} & \quad \text{The minimum } V_C \text{ is limited by } Q_1 \text{ when it enters saturation} \\
V_{CE} = V_c - V_E = 19.3 - I_{c,\text{max}} R_c - I_{c,\text{max}} R_c \frac{R_c}{\alpha} R_E = 19.3 - I_{c,\text{max}} R_c (1 + \frac{101}{100} \times \frac{3}{4}) \geq 0.3 \\
\Rightarrow I_{c,\text{max}} R_c \leq 10.81V \\
\Rightarrow V_{C_{\text{min}}} = V_{CC} - V_D - I_{c,\text{max}} R_c = -1.51V \\
\text{The maximum } V_C \text{ is limited by the power supply, i.e., } V_{C_{\text{max}}} = V_{CC} - V_D = 9.3V \\
\text{Since the collector is biased at 0 V, to get a symmetric and unclipped sinusoid, the maximum possible amplitude is } \min\{|V_{C_{\text{min}}}|, |V_{C_{\text{max}}}|\} = 1.51V.
\end{align*}
Problem 4 (20 Points)

For the common emitter amplifier circuit shown below, assume for the transistor, $\beta = 100$.

(a) Find the dc collector current, and the dc voltages at the emitter, base and collector.

(b) Find $r_e$, $r_p$ and $g_m$.

(c) Find the input resistance ($R_{in}$), the voltage gain ($v_{out}/v_{in}$), and the output resistance ($R_{out}$).

(a) \[ 3.5V = I_B R_B + V_{BE} + (1 + \beta) I_B R_E \Rightarrow 2.8 = I_B \left[ R_B + (1 + \beta) R_E \right] \]

\[ \Rightarrow I_B = \frac{2.8}{\frac{13 \times 7}{13 + 7} + 101 \times 1.8} = 0.015mA \]

\[ \Rightarrow I_C = \beta I_B = 1.5mA \]

\[ V_C = V_{cc} - I_C R_C = 10 - 1.5 \times 3.5 = 4.74V \]

\[ V_B = 3.5 - I_B R_B = 3.5 - 0.015 \times 4.55 = 3.43V \]

\[ V_E = V_B - V_{BE} = 3.43 - 0.7 = 2.73V \]

(b) \[ r_x = \frac{V_T}{I_B} = \frac{25mV}{0.015mA} = 1.667k\Omega \]

\[ r_e = \frac{V_T}{I_E} = \frac{25mV}{101 \times 0.015mA} = 16.50\Omega \]

\[ g_m = \frac{I_C}{V_T} = \frac{1.5mA}{25mV} = 60mA/V \]

(c)
Problem 5 (20 Points)

For the amplifier circuit shown below, assume that the transistor has a $\beta = 100$ and $V_A = 100$ V.

(a) Find the dc voltages at the base, emitter and collector terminals of the transistor.

(b) Find $g_m$, $r_\pi$, $r_e$ and $r_o$.

(c) Find the voltage gain at node B ($v_{out1}/v_{in}$), when node A is connected to the +10V supply.

(d) Also find the voltage gain at node A ($v_{out2}/v_{in}$), when node B is connected to the -10V supply.

\[ v_b = i_b r_e + (1 + \beta) i_e R_E \]
\[ R_{ib} = v_b / i_b = r_e + (1 + \beta) R_E = 1.667 + 101 \times 0.2 = 21.867 \Omega \]
\[ R_{in} = R_B || R_{ib} = (4.55 \times 21.87) / (4.55 + 21.87) = 3.766 \Omega \]
\[ v_{out} = -\beta i_b R_C = -\beta (v_b / R_{ib}) R_C \]
\[ A_v \equiv v_{out} / v_b = -\beta (R_C / R_{ib}) = -100(3.5 / 21.87) \approx 16.0 \]
\[ G_v \equiv v_{out} / v_{in} = A_v R_{in} / (R_{in} + R_b) = -16 \times 3.766 / (3.766 + 5) \approx -6.87 \]
\[ R_{out} = R_C = 3.5 k\Omega \]

(a) $I_E = 1 mA \Rightarrow I_B = I_E / (1 + \beta) = 1 mA / 101 = 9.99 \mu A$
\[ V_B = I_B R_B = 9.99 \mu A \times 10 k\Omega = 0.099 V \]
\[ V_E = V_B + V_{BE} = 0.099 + 0.7 = 0.799 V \]
\[ V_C = -V_{CE} + \alpha I_E R_C = -10 + 1 \times (100 / 101) \times 5 = -5.05 V \]

(b) \[ g_m = \frac{I_C}{V_T} = \frac{100}{101 \times 25} = 39.6 mA / V \quad , \quad r_x = \frac{V_T}{I_B} = \frac{25}{9.9 \times 10^{-3}} = 2.525 k\Omega \quad , \]
\[ r_o = \frac{V_I}{I_C} = \frac{100}{100 / 101} = 10 k\Omega \]
\( R_{\text{sh}} = r_x = 2.525k\Omega \)

\( R_{\text{in}} = R_B \parallel r_x = (10 \times 2.525)/(10 + 2.525) = 2.016k\Omega \)

\( i_b = -v_b / R_B \)

\( v_{\text{out1}} = \beta i_b (R_C \parallel r_o) = -\beta (v_b / R_B) (R_C \parallel r_o) \)

\[ \Rightarrow A_{\text{v1}} \equiv v_{\text{out1}} / v_b = -\beta \frac{R_C \parallel r_o}{R_B} = -100 \frac{5 \times 100}{2.525} = -188.59 \]

\[ \Rightarrow G_{\text{v1}} \equiv \frac{v_{\text{out1}}}{v_{\text{in}}} = A_{\text{v1}} \frac{R_{\text{m}}}{R_{\text{m}} + R_i} = -188.59 \times \frac{2.016}{2.016 + 5} = -54.19 \]

\( v_{\text{out2}} = (1 + \beta) i_b r_o, \quad v_b = v_{\text{out2}} + i_b r_x = i_b \left[ r_x + (1 + \beta) r_o \right] \)

\[ \Rightarrow A_{\text{v2}} = \frac{v_{\text{out2}}}{v_b} = \frac{v_{\text{out2}}}{i_b} \times \frac{i_b}{v_b} = \frac{(1 + \beta) r_o}{r_x + (1 + \beta) r_o} \approx 1 \]

\[ \Rightarrow R_{\text{sh}} = \frac{v_b}{i_b} = r_x + (1 + \beta) r_o \approx 10.2M\Omega \]

\[ \Rightarrow R_{\text{in}} = R_B \parallel R_{\text{sh}} = \left( \frac{1}{10} + \frac{1}{10200} \right)^{-1} \approx 10k\Omega \]

\[ \Rightarrow G_{\text{v2}} \equiv \frac{v_{\text{out2}}}{v_{\text{in}}} = A_{\text{v2}} \frac{R_{\text{in}}}{R_{\text{in}} + R_s} = 1 \times \frac{10}{10 + 5} = 0.667 \]