Q1

(a)  
\[ \begin{align*}
V_x^- & \quad I_x \quad R_1 \quad D_1 \quad V_x^+ \\
& \quad -V_b
\end{align*} \]

Case 1: \( V_b = -1 \text{V} \)

when \( V_x < -1 \text{V} \), \( D_1 \) turns off
\[ I_x = I_{R_1} = 0 \]

when \( V_x > -1 \text{V} \), \( D_1 \) turns on
\[ I_x = I_{R_1} = \frac{V_x - V_b}{R_1} = \frac{V_x + 1}{R_1} \]

Case 2: \( V_b \geq 1 \text{V} \)

when \( V_x \leq 1 \text{V} \), \( D_1 \) off
\[ I_x = I_{R_1} = 0 \]

when \( V_x > 1 \text{V} \), \( D_1 \) on
\[ I_x = I_{R_1} = \frac{V_x - 1}{R_1} \]

(b)  
\[ \begin{align*}
V_x^- & \quad I_x \quad I_{R_1} \quad R_1 \quad V_x^+ \\
& \quad I_{R_2} \quad -V_b
\end{align*} \]

Case 1: \( V_b = -1 \text{V} \)

\[ I_{R_1} = \frac{V_x - V_b}{R_1} \]

\[ \begin{align*}
I_{R_2} = \begin{cases} 
0 & \text{D}_1 \text{ off} \\
\frac{V_x}{R_2} & \text{D}_1 \text{ on}
\end{cases}
\end{align*} \]

\[ I_x = I_{R_1} + I_{R_2} \]

Case 2: \( V_b = 1 \text{V} \)

\[ I_{R_1} = \frac{V_x - 1}{R_1} \]

\[ \begin{align*}
I_{R_2} = \begin{cases} 
0 & \text{D}_1 \text{ off} \\
\frac{V_x}{R_2} & \text{D}_1 \text{ on}
\end{cases}
\end{align*} \]

\[ I_x = \frac{V_x - 1}{R_1} + \frac{V_x}{R_2} \]

\[ I_x = \frac{V_x - 1}{R_2} \]
Since the voltage difference between node 0 and node 2 is always equal to $V_B$, no matter $D_1$ turns on or off.

$I_X = I_{R_1} = \frac{V_X - V_B}{R_1}$

**Case 1: $V_B = -1 V$**

$I_X = I_{R_1} = \frac{V_X + 1}{R_1}$

**Case 2: $V_B = +1 V$**

$I_X = I_{R_1} = \frac{V_X - 1}{R_1}$
Q2 (a) Assume the voltage to turn on diode is $V_D$

1. When $V_{in}$ is negative infinity, intuitively, $D_1$ is off. $I_{R_1} = 0$.

Assume $D_2$ is on, then $V_{out} = -V_D$.

$$I_{R_2} = \frac{V_D - (-V_D)}{R_2} = \frac{V_D}{R_2}.$$  
Since $I_{D_2} > 0$, so $I_{R_2} + I_{R_1} + I_{D_2} > \frac{V_D}{R_2}$ does not satisfy KCL, so the assumption $D_2$ is on is wrong.

Then $D_2$ is off. $I_{D_2} = 0$.

According to KCL, $I_{R_1} + I_{R_2} + I_{D_2} = 0$.

So, $I_{R_2} = 0$, then $V_{out} = 0$.

2. When $V_{in} \geq V_D$, $D_1$ turns on. $I_{R_1} > 0$.

Assume $D_2$ is on, then $V_{out} = -V_D$.

$$I_{R_1} = \frac{V_D}{R_2}, \quad I_{D_2} \geq 0.$$  
So, $I_{R_1} + I_{R_2} + I_{D_2} > 0$. Doesn't satisfy KCL.

So, $D_2$ is off.

So, $I_{R_1} = \frac{V_{in} - V_D}{R_1 + R_2}$  
$V_{out} = I_{R_1} \cdot R_2 = \frac{R_2}{R_1 + R_2} (V_{in} - V_D)$.

So, $V_{in} < V_D$  
$V_{in} \geq V_D$

$I_{R_1} = 0$  
$V_{out} = 0$

$I_{R_1} = \frac{V_{in} - V_D}{R_1 + R_2}$  
$V_{out} = \frac{R_2}{R_1 + R_2} (V_{in} - V_D)$.
(b) \[ \begin{align*}
&\text{when } V_{in} \text{ in negative infinity, intuitively, } D_1 \text{ is off }, D_2 \text{ is on.} \\
&\text{Then current } -I_{R_1} = \frac{V_D - V_{in}}{R_1 + R_2} \Rightarrow I_{R_1} = \frac{V_D + V_{in}}{R_1 + R_2}
\end{align*} \]

when \( V_{in} \) increases until \( V_{in} = -V_D \), \( I_{R_1} = 0 \), \( V_{out} = -V_D + I_{R_2} R_2 \)

Di is on the edge of turning on and off.

So, when \( V_{in} = -V_D \), \( D_2 \) turns off.

\( \Box \)

\[ V_D = V_{in} \]

\[ I_{R_1} = V_D \]

\[ V_{out} = \frac{R_2}{R_1 + R_2} V_{in} - \frac{R_1}{R_1 + R_2} V_D \]

\( \therefore \) \( V_{in} < -V_D \)

\[ I_{R_1} = \frac{V_D + V_{in}}{R_1 + R_2} \]

\[ V_{out} = \frac{R_2}{R_1 + R_2} V_{in} - \frac{R_1}{R_1 + R_2} V_D \]

\( \therefore \) \( V_{in} \geq -V_D \)

\( I_{R_1} = 0 \)

\( V_{out} = V_{in} \)
(C) \[ \begin{align*}
V_{in} & \rightarrow D \rightarrow V_{out} \\
V_{r} & \rightarrow I_{R1} \rightarrow V_{o} \\
R_{1} & \rightarrow I_{R2} \rightarrow I_{D2} \\
R_{2} & \rightarrow V_{e} = 1.5 V \\
\end{align*} \]

1. When \( V_{in} \) is negative and \( \infty \), intuitively, \( D_1 \) turns off, \( D_2 \) on, \( V_{out} \) always equal to \(-V_{d}\).

\[ I_{R1} = \frac{V_{in} - V_{out}}{R_{1}} = \frac{V_{in} + V_{d}}{R_{1}} \]

\[ I_{R2} = \frac{V_{e} - V_{out}}{R_{2}} = \frac{1.5 + V_{d}}{R_{2}} \]

Based on KCL, \( I_{R1} + I_{R2} + I_{D2} = 0 \).

When \( V_{in} \) keeps increasing so that \( D_2 \) at the edge of turning on and off. At the edge point, \( I_{D2} = 0 \).

So.

\[ I_{R1} + I_{R2} = \Rightarrow \frac{V_{in} + V_{d}}{R_{1}} = \frac{1.5 + V_{d}}{R_{2}} \]

\[ V_{in} = \frac{R_{1}}{R_{2}} (1.5 - \left( \frac{R_{1}}{R_{2}} + 1 \right)) V_{d} \]

2. When \( V_{in} \geq - \frac{R_{1}}{R_{2}} (1.5 - \left( \frac{R_{1}}{R_{2}} + 1 \right)) V_{d} \), \( D_1 \) and \( D_2 \) turn off.

\[ I_{R1} = I_{R2} = \frac{V_{in} - 1.5}{R_{1} + R_{2}} \]

When \( V_{in} \) keeps increasing, \( D_1 \) turns on.

At the edge of \( D_1 \), turning on and off. \( V_{in} = V_{out} + V_{d} \).

Before \( D_1 \) turns on. \( I_{R1} = \frac{V_{in} - 1.5}{R_{1} + R_{2}} \), \( V_{out} = V_{in} - I_{R1} \cdot R_{1} \)

At the point at \( D_1 \) turns on. \( I_{R1} \cdot R_{1} = V_{d} \).

\[ \frac{V_{in} - 1.5}{R_{1} + R_{2}} \cdot R_{1} = V_{d} \Rightarrow V_{in} = 1.5 + \frac{R_{1} + R_{2}}{R_{1}} V_{d} \]

3. When \( V_{in} \geq 1.5 + \frac{R_{1} + R_{2}}{R_{1}} V_{d} \), \( D_1 \) turns on, \( D_2 \) off.

\[ I_{R1} = \frac{V_{d}}{R_{1}} \]

\[ V_{out} = V_{in} - V_{d} \]

So, when \( V_{in} < - \frac{R_{1}}{R_{2}} (1.5 - \left( \frac{R_{1}}{R_{2}} + 1 \right)) V_{d} \).

\[ I_{R1} = \frac{V_{in} + V_{d}}{R_{1}} \]

\[ V_{out} = -V_{d} \]

When \(- \frac{R_{1}}{R_{2}} (1.5 - \left( \frac{R_{1}}{R_{2}} + 1 \right)) V_{d} \leq V_{in} < 1.5 + \frac{R_{1} + R_{2}}{R_{1}} V_{d} \)

\[ I_{R1} = \frac{V_{in} - 1.5}{R_{1} + R_{2}} \]

\[ V_{out} = V_{in} - \frac{R_{1} + R_{2}}{R_{1} + R_{2}} \]

When \( V_{in} \geq 1.5 + \frac{R_{1} + R_{2}}{R_{1}} V_{d} \)

\[ I_{R1} = \frac{V_{d}}{R_{1}} \]

\[ V_{out} = V_{in} - V_{d} \]

So, when \( V_{in} < - \frac{R_{1}}{R_{2}} (1.5 - \left( \frac{R_{1}}{R_{2}} + 1 \right)) V_{d} \).

\[ I_{R1} = \frac{V_{in} + V_{d}}{R_{1}} \]

\[ V_{out} = -V_{d} \]

When \(- \frac{R_{1}}{R_{2}} (1.5 - \left( \frac{R_{1}}{R_{2}} + 1 \right)) V_{d} \leq V_{in} < 1.5 + \frac{R_{1} + R_{2}}{R_{1}} V_{d} \)

\[ I_{R1} = \frac{V_{in} - 1.5}{R_{1} + R_{2}} \]

\[ V_{out} = V_{in} - \frac{R_{1} + R_{2}}{R_{1} + R_{2}} \]

When \( V_{in} \geq 1.5 + \frac{R_{1} + R_{2}}{R_{1}} V_{d} \)

\[ I_{R1} = \frac{V_{d}}{R_{1}} \]

\[ V_{out} = V_{in} - V_{d} \]
when Rs swap as shown in the figure.

The circuit works as following:

when Vin is positive, the circuit is like as following:

So, all the patches are broken. Then No current path exist.
So, Void doesn't charge with Vin.

When Vin is negative, the circuit is as following:
the current will flow as shown, again Void doesn't follow Vin.

So, when Rs swap, the circuit can not be rectifier.

Assume use constant voltage model.
Assume using constant voltage model

When \( V_{in} \) is positive, circuit will be shown as in figure
\[
V_{act} = V_{in} - V_o
\]

When \( V_{in} \) is negative, current will flow as shown in the following figure, no current goes through \( R_4 \), so \( V_{act} = 0 \)

So, it is not full-wave rectifier.

Notes:
For Q.3 and Q.4, if you assume ideal model for diodes, that's fine. The only difference is that the voltage across diode when it's on is 0 instead of \( V_o \).