During our last three weeks, we have examined several methods for replacing complex resistor circuit constructions with simplified equivalent circuits, for example series or parallel equivalent circuits.

Also, we saw an example of a source transformation, the $\Delta$ to $Y$ and $Y$ to $\Delta$ transformations of resistor circuits.

Many amplifier, interface circuit, and power source systems are described effectively in terms of equivalent circuits where internal voltage sources, current sources, and resistors are replaced by a simplified circuit.

We are particularly interested in circuits that are presented to us as two terminal circuit elements and we are interested in characterizing the circuit behavior at the two terminals.

We would like to replace all of the complexity of the internal circuit structure with either an equivalent voltage or current source and appropriate equivalent resistors.

The motivation for equivalent circuits is to enable rapid calculation, circuit understanding and design intuition.

Let's consider the voltage that is produced by an arbitrary arrangement of linear resistors and sources.

Superposition states that the voltage across any terminal pair is a linearly-weighted sum of voltage contributions from all elements.
• Lets consider a terminal pair across which the voltage is V. Then,
• We can separate all of the terms in the defining equations to be a sum of voltages due to voltage sources and a sum of voltages due to resistances and currents.

\[ V = \sum V_k + \sum I_l R_l \]

All Sources, k  All Resistors, l

• The first term is referred to as an Equivalent Voltage Source.
• Now, if we were to set all sources to zero, and then apply a voltage V, to the two terminals, then a current, I would flow.
• This current I, is an Equivalent Current source and the resistance, V/I forms an Equivalent Resistance.
• Our circuit with its two terminals, may be represented, therefore by one of two alternatives Equivalent Circuits:

![Diagram](image)

Figure 1. Replacement of a Linear Circuit System with either a Voltage Source Equivalent or Current Source Equivalent Circuit containing an Equivalent Resistor
The observation that any circuit of linear sources and passive elements may be replaced by equivalent elements was first made by Helmholtz in 1853.

Thevenin, a French telegraph engineer, drew attention to the circuit on the left hand of Figure 6. This equivalent circuit is now called the Thevenin Equivalent.

Edward Norton, an engineer at Bell Laboratories, is credited with the Norton Equivalent circuit shown at the right of Figure 6.

Summary:

- The Thevenin and Norton Equivalent Circuits will replace a linear two terminal circuit structure with simply either:
  - For a Thevenin Equivalent: A Thevenin Equivalent Voltage Source, $V_{Th}$, in series with a Thevenin Equivalent Resistance $R_{Th}$.
    - Also, for all circuits, $R_N = R_{Th}$
DERIVING THEVENIN AND NORTON EQUIVALENT CIRCUITS

- We may define a procedure for determining the values of Equivalent Sources and Resistances. These will be our procedures:

**PROCEDURES FOR THEVENIN EQUIVALENT CIRCUITS**

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance $R_{AB}$. This resistance will equal $R_{TH}$

4) Return all Independent Sources to their original values and compute the voltage value corresponding to the voltage drop from A to B, $V_{AB}$ (with Nodes A and B open-circuited). This voltage is the Thevenin Equivalent voltage, $V_{AB} = V_{Th}$

5) You may compute this voltage using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Thevenin Equivalent Circuit.

**PROCEDURES FOR NORTON EQUIVALENT CIRCUITS**

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance $R_{AB}$. This resistance will equal $R_{N}$

4) Return all Independent Sources to their original values and then apply a short circuit at the terminals A and B. Compute the current value from A to B, $I_{AB}$ (with Nodes A and B short-circuited). This current is the Norton Equivalent Current, $I_{AB} = I_{N}$

5) You may compute this current using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Norton Equivalent Circuit using this $I_{N}$ and $R_{N}$. Note polarities for $I_{N}$. 
• Lets compute a Thevinin and Norton Source Equivalent for a circuit.

![The Example Circuit](image)

*Figure 2. The Example Circuit*

• Here is the procedure:

**PROCEDURES FOR THEVENIN EQUIVALENT CIRCUITS**

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.
2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.
3) Compute the resistance $R_{AB}$. This resistance will equal $R_{TH}$.
4) Return all Independent Sources to their original values and compute the voltage value corresponding to the voltage drop from A to B, $V_{AB}$ (with Nodes A and B open-circuited). This voltage, $V_{AB} = V_{TH}$.
5) You may compute this voltage using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.
6) Draw the new Thevenin Equivalent Circuit.

• Lets follow this. First, we see that the terminals are labeled.
• Lets remove the voltage source and replace it with its “zero” element – a short circuit.
• Then, lets compute $R_{AB}$.
• This is simple, $R_{AB}$ will be a resistance consisting of an $8\Omega$ resistor in series with a parallel combination of a $10\Omega$ and $40\Omega$ resistor. This parallel combination is itself, $8\Omega$, so $R_{AB} = R_{TH} = 16\Omega$.
• Now, lets compute $V_{TH}$.
• We return the voltage source to its original value, and compute the voltage at the terminals, a and b.
• Here, we can just use the Voltage Divider equation. Why?
• Now, \( V_{AB} = V_{TH} = 48 \text{V} \). Thus, our equivalent circuit is:

\[\begin{align*}
\text{Figure 3. The Example Circuit and its Thevenin Equivalent} \\
\text{PROCEDURES FOR NORTON EQUIVALENT CIRCUITS}
\end{align*}\]

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance \( R_{AB} \). This resistance will equal \( R_N \).

4) Return all Independent Sources to their original values and then apply a short circuit at the terminals A and B. Compute the current value from A to B, \( I_{AB} \) (with Nodes A and B short-circuited). This current is the Norton Equivalent Current, \( I_{AB} = I_N \).

5) You may compute this current using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.)

6) Draw the new Norton Equivalent Circuit using this \( I_N \) and \( R_N \). Note polarities for \( I_N \).

• Lets follow this. First, the terminals are labeled.

• We remove the voltage source and replace it with its “zero” element – a short circuit.

• Then, we compute \( R_{AB} \).

• We just found this to be \( R_{AB} = R_N = 16 \Omega \).

• Now, lets compute \( I_N \).

• We return the voltage source to its original value, and compute the short circuit current value at the terminals, a and b. Lets examine the circuit that results from this:
• Let's solve for the current flowing through the 8Ω resistor. This is a familiar problem. Let's just compute the voltage drop across the 8Ω resistor. This is just equal to $v_c$.

• First, we can compute the voltage at the node joining the 40Ω and 8Ω resistors. We start by replacing them with their parallel equivalent. This parallel equivalent is just 6.66Ω.

• Now, then we can use the Voltage Divider Equation again. The voltage, $v_c$, from this is,

\[ v_c = 60V \frac{6.66\Omega}{16.66\Omega} = 24V \]

• Then, the current through the 8Ω resistor is just $24V / 8Ω = 3A$

• Thus, $I_N = 3A$. Thus, our equivalent circuit is:

**Figure 4. Circuit construction for computing the Norton Equivalent Current**

**Figure 5. The Norton Source Equivalent for our Example Circuit**

• Let's check this result and our understanding.

• First, these two circuits should show the same open circuit output voltage:
The Thevenin Source Equivalent output voltage is 48V – this is easily seen.

The Norton Source Equivalent is also 48V, just the voltage drop due to the 3A current flow through the 16Ω resistor.

Also, these circuits should show the same short circuit output current.

The Thevenin Source Equivalent short circuit current is 3A

The Norton Source Equivalent short circuit current is 3A

These three circuits, the original circuit and the two Thevenin and Norton Equivalents all show exactly the same terminal characteristics.

SOURCE EQUIVALENT CIRCUITS WITH DEPENDENT SOURCES

In the development of amplifier circuits, we will frequently encounter the need for developing Source Equivalent circuits involving dependent sources.

All of our techniques will apply.

However, we must introduce one new method for computing resistance. This will be useful to you later, for example in EE115B.

Here is our circuit (a textbook Drill Exercise):

![An Example Circuit for computation of Source Equivalents](image)

**Figure 6. An Example Circuit for computation of Source Equivalents**

First, let's use the general procedures to compute the Thevenin Source Equivalent.

We will begin by computing $V_{TH}$

Let's use the Node Voltage method with a Reference Node where the two Independent
Sources and the 8Ω resistor meet.

- Now, note that there is only one Node Equation required. Why?
- At terminal a, an Essential Node, we have the KCL equation:
  \[(24V - v_a)/2 + (-3i_x) - (4A) - v_a/8 = 0\]
- Also, the Dependent Source constraint equation is:
  \[i_x = v_a/8\]
- This is now easy to solve. Substituting for \(i_x\).
  \[(24V - v_a)/2 + (-3v_a/8) - (4A) - v_a/8 = 0\]
- and
  \[v_a = v_{TH} = 8V\]
- Now, we must compute \(R_{AB}\)
- There are two approaches:
  - 1) One may construct a method where we short circuit the output terminals and compute current in the short circuit, \(i_{SC}\)
    - Then, the resistance, \(R_{AB}\), is just \(v_{TH}/i_{SC}\)
    - Lets return to our definitions of the Thevenin Equivalent to see this.
  - 2) An alternative (and our recommend) approach for problems we will encounter, is to compute \(R_{AB}\) in a fundamental method:
    - Replace all Independent Sources with their “zero” elements. Note, Dependent Sources remain.
    - Apply a test voltage, \(V_T\) to the output terminals
    - This will yield a test current, \(I_T\)
    - Compute \(R_{AB} = V_T/I_T\)
    - This is a powerful method that will never let you down.
- Let us proceed with the second case
- Let us replace the all Independent Sources with their “zero” elements. Also, lets apply the voltage, \(V_T\).
Let us again use a Node Voltage method to compute $I_T$.

Again, we will place our Reference at the lower node on this diagram where the $8\Omega$, $2\Omega$, and the two sources meet.

Then, the Node Voltage Equation is:

$$\frac{(-V_T)}{2\Omega} + (-3i_x) - \frac{V_T}{8\Omega} + I_T = 0$$

but,

$$i_x = \frac{V_T}{8}$$

So, our equation becomes:

$$\frac{(-V_T)}{2\Omega} + (-3\frac{V_T}{8\Omega}) - \frac{V_T}{8\Omega} + I_T = 0$$

Rearranging,

$$-\frac{V_T}{1\Omega} + I_T = 0$$

But, by definition:

$$R_{AB} = \frac{V_T}{I_T} = 1\Omega$$

Thus, our Thevenin Source Equivalent circuit is
Point for discussion: note that the Thevenin resistance is a very low value. Why? This has application in low input impedance amplifiers.