Grading Policy

- Home Works    10%
- 1\textsuperscript{st} Midterm    20% (May 2)
- 2\textsuperscript{nd} Midterm   30% (May 23)
- Final        40%
Topics to be covered

Course reader: Fundamentals of Microelectronics (Part I)

- CH1  Why Microelectronics? (Week 1)
- CH2  Basic Physics of Semiconductors (Week 1)
- CH3  Diode Circuits (Week 2)
- CH4  Physics of Bipolar Transistors (Week 3)
- CH5  Bipolar Amplifiers (Week 4-6)
- CH6  Physics of MOS Transistors (Week 7)
- CH7  CMOS Amplifiers (Week 8-9)
Chapter 1  Why Microelectronics?
1.1 Electronics versus Microelectronics
1.2 Example of Electronic System: Cellular Telephone
1.3 Analog versus Digital
Cellular Technology

- An important example of microelectronics.
- Microelectronics exists in black boxes that process the received and transmitted voice signals.
Frequency Up-conversion

- Frequency is up-converted by multiplying two sinusoids.
- When multiplying two sinusoids in time domain, their spectrums are convolved in frequency domain.
Two frequencies are multiplied and radiated by an antenna in (a).

A power amplifier is added in (b) to boost the signal.
High frequency is down-converted to baseband by multiplying with itself.

A low-noise-amplifier is needed for signal boosting without creating extra noise.
Digital or Analog?

- $X_1(t)$ is operating at 100Mb/s and $X_2(t)$ is operating at 1Gb/s.
- A digital signal operating at very high frequency is very “analog”.
Chapter 2  Basic Physics of Semiconductors

- 2.1 Semiconductor materials and their properties
- 2.2 PN-junction diodes
- 2.3 Reverse Breakdown
**Semiconductor Physics**

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<th>Semiconductors</th>
<th>PN Junction</th>
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- Microelectronics is made of semiconductor devices
- PN junction is the most fundamental semiconductor device.
To understand PN junction’s I-V characteristics, it is important to understand charge carriers’ behavior in solids, carrier density modulations through biasing and various types of charge transportation.
Periodic Table

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<tr>
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<th>III</th>
<th>IV</th>
<th>V</th>
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<tbody>
<tr>
<td>Boron (B)</td>
<td></td>
<td>Carbon (C)</td>
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<tr>
<td>Aluminum (Al)</td>
<td>Silicon (Si)</td>
<td>Phosphorus (P)</td>
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<tr>
<td>Galium (Al)</td>
<td>Germanium (Ge)</td>
<td>Arsenic (As)</td>
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- This table contains elements with three to five valence electrons, with Si being the most important semiconductor material.
Si has four valence electrons. Therefore, it can form covalent bonds with four of its neighbors.

When temperature goes up, electrons in the covalent bond can become free.
Electron-Hole Pair Interaction

- With free electrons breaking off covalent bonds, holes are generated.
- Holes can be filled by absorbing other free electrons, so effectively there is a flow of charge carriers.
Free Electron Density at a Given Temperature

\[ n_i = 5.2 \times 10^{15} T^{3/2} \exp \left( \frac{-E_g}{2kT} \right) \text{ electrons / cm}^3 \]

\[ n_i (T = 300K) = 1.08 \times 10^{10} \text{ electrons / cm}^3 \]

\[ n_i (T = 600K) = 1.54 \times 10^{15} \text{ electrons / cm}^3 \]

- \( E_g \), or bandgap energy determines how much effort is needed to break off an electron from its covalent bond.
- There exists an exponential relationship between the free-electron density and bandgap energy and temperature.
- A pure Si can be doped with other elements to change its electrical property.
- For example, if Si was doped with P (phosphorous), then it would have more electrons, or become type N (electron).
If Si was doped with B (boron), then it would have more holes, or become type P.
Summary of Charge Carriers

Intrinsic Semiconductor

Extrinsic Semiconductor

Silicon Crystal
$N_D$ Donors/cm$^3$

$n$-Type Dopant (Donor)

Free Majority Carrier

Silicon Crystal
$N_A$ Acceptors/cm$^3$

$p$-Type Dopant (Acceptors)

Free Majority Carrier
Electron and Hole Densities

\[ np = n_i^2 \]

\[ p \approx N_A \]
\[ n \approx \frac{n_i^2}{N_A} \]
\[ n \approx N_D \]
\[ p \approx \frac{n_i^2}{N_D} \]

- The product of electrons and holes densities is ALWAYS equal to the square of intrinsic electron density regardless of doping state.
The process in which charge particles move because of an electric field is called drift.

Charge particles will move in a constant velocity that is proportional to the electric field.

\[ v_h = \mu_p E \]

\[ v_e = -\mu_n E \]
Electric current is calculated as the amount of charges per unit time that passes thru a cross-section if the charges travel with a velocity of $v$ cm/s.

\[ I = -v \cdot W \cdot h \cdot n \cdot q \]
Current Flow: Drift

\[ J_n = \mu_n E \cdot n \cdot q \]
\[ J_{tot} = \mu_n E \cdot n \cdot q + \mu_p E \cdot p \cdot q \]
\[ = q(\mu_n n + \mu_p p)E \]

- Since velocity is equal to \( \mu E \), drift characteristic is shown by substituting \( V \) with \( \mu E \) in the general current equation.
- The total Current density contains both electrons and holes.
Velocity Saturation

In reality, velocity does not increase linearly with electric field. It will eventually saturate to a critical value.

\[ \mu = \frac{\mu_0}{1 + bE}, \quad v_{sat} = \frac{\mu_0}{b}, \quad v = \frac{\mu_0}{1 + \frac{\mu_0 E}{v_{sat}}} E \]
Another type of charge transportation is known as diffusion, in which charge particles will move from a region of high concentration to a region of low concentration. It is analogous to an everyday example of an ink droplet in water.
Current Flow: Diffusion

\[ I = A q D_n \frac{dn}{dx} \]

\[ J_p = -q D_p \frac{dp}{dx} \]

\[ J_n = q D_n \frac{dn}{dx} \]

\[ J_{tot} = q \left( D_n \frac{dn}{dx} - D_p \frac{dp}{dx} \right) \]

- Diffusion current is proportional to the gradient of charge along the direction of current flow.
- Its total current density is consisted of both electrons and holes.
Linear charge density profile means constant diffusion current, whereas nonlinear charge density profile means non-constant diffusion current.
While the underlying physics behind drift and diffusion currents are totally different, Einstein’s relation provides a mysterious link between the two.
When N-type and P-type dopants are introduced side-by-side in a semiconductor, a PN junction, or diode is formed.
In order to understand the operation of a diode, it is necessary to study its three operation regions: equilibrium, reverse bias, and forward bias.
Because each side of the junction contains an excess of holes or electrons than the other, there exists a large concentration gradient. Therefore, two large diffusion currents will flow across the junction from each side.
Depletion Region

- As free electrons and holes diffuse across the junction, a region of fixed ions are left behind. This region is known as the depletion region.
As mentioned earlier, the depletion region contains fixed ions. These fixed ions will create an electric field that results in a drift current.
At equilibrium, the drift current flowing from one direction will cancel out the diffusion current flowing from the opposite direction, creating a net current of zero.

The figure on the left shows the charge profile of the PN junction at equilibrium.
Built-in Potential

\[ q\mu_p p E = -qD_p \frac{dp}{dx} \]

\[-\mu_p p \frac{dV}{dx} = -D_p \frac{dp}{dx} \]

\[ \mu_p \int_{x_1}^{x_2} dV = D_p \int_{p_p}^{p} \frac{dp}{p} \]

\[ V(x_2) - V(x_1) = D_p \ln \frac{p_p}{\mu_p p_n} \]

\[ V_0 = \frac{kT}{q} \ln \frac{p_p}{p_n}, V_0 = \frac{kT}{q} \ln \frac{N_A N_D}{n_i^2} \]

- Because of the electric field across the junction, there exists a built-in potential. Its derivation is shown above.
When the N-type region of a diode is connected to a higher potential than the P-type region, the diode is under reverse bias.

A wider depletion region and larger built-in electric field across the junction are the results of reverse bias.
Reverse Biased Diode’s Application: Voltage-Dependent Capacitor

- There exists a charge vs. reverse bias voltage relationship in the PN junction. Thus it can be viewed as a capacitor.
- By varying VR, the depletion width changes, changing its capacitance value, therefore the PN junction is actually a voltage-dependent capacitor.
Voltage-Dependent Capacitance

The equations that describe the voltage-dependent capacitance is shown above, and as we can see it indeed changes with voltage.

\[ C_j = \frac{C_{j0}}{\sqrt{1 + \frac{V_R}{V_0}}} \]

\[ C_{j0} = \sqrt{\frac{\varepsilon_{si} q \frac{N_A N_D}{2 N_A + N_D}}{V_0}} \]

- The equations that describe the voltage-dependent capacitance is shown above, and as we can see it indeed changes with voltage.
A very important application of a reverse-biased PN junction is VCO. In which a LC tank is used as an oscillator. By changing $V_R$, we can change C, which also changes the oscillation frequency.

\[ f_{res} = \frac{1}{2\pi} \frac{1}{\sqrt{LC}} \]
When the N-type region of a diode is at a lower potential than the P-type region, the diode is forward biased.

The depletion width is shortened and the built-in electric field decreased.
Under forward bias, minority carriers in each region will increase due to the lowering of built-in field/potential. Therefore, diffusion currents will increase to supply these minority carriers.

\[
\begin{align*}
    p_{n,e} &= \frac{p_{p,e}}{\exp \frac{V_0}{V_T}} \\
    p_{n,f} &= \frac{p_{p,f}}{\exp \frac{V_0 - V_F}{V_T}}
\end{align*}
\]
Diffusion Current in Forward Bias

\[ \Delta n_p \approx \frac{N_D}{V_0} \frac{V_F}{V_T} \left( \exp \frac{V_F}{V_T} - 1 \right) \]

\[ \Delta p_n \approx \frac{N_A}{V_0} \frac{V_F}{V_T} \left( \exp \frac{V_F}{V_T} - 1 \right) \]

\[ I_{tot} \propto \frac{N_A}{V_0} \frac{V_F}{V_T} \left( \exp \frac{V_F}{V_T} - 1 \right) + \frac{N_D}{V_0} \frac{V_F}{V_T} \left( \exp \frac{V_F}{V_T} - 1 \right) \]

\[ I_{tot} = I_s \left( \exp \frac{V_F}{V_T} - 1 \right) \]

\[ I_s = Aq n_i^2 \left( \frac{D_n}{N_A L_n} + \frac{D_p}{N_D L_p} \right) \]

- As explained earlier, diffusion current will increase in order to supply the increase in minority carriers. The mathematics are shown above.
Minority Charge Gradient

- Minority charge profile should not be constant along the x-axis, because there will not be a concentration gradient and hence diffusion current will be zero.
- Recombination of the minority carriers with the majority carriers accounts for the dropping of minority carriers as they go deep into the P or N region.
To summarize, under forward bias condition, there are large diffusion currents of minority carriers around the junction. However, as we go deep into the P and N regions, recombination currents from the majority carriers dominate. These two currents add up to a constant value.
IV Characteristic of PN Junction

The current and voltage relationship of a PN junction is exponential in forward bias region, and essential constant in reverse bias region.

\[ I_D = I_S \left( \exp \frac{V_D}{V_T} - 1 \right) \]

- The current and voltage relationship of a PN junction is exponential in forward bias region, and essential constant in reverse bias region.
Since junction currents are proportional to the junction’s cross-sectional area. Two PN junctions put in parallel is effectively one PN junction with twice the cross-sectional area, and hence twice the current.
A diode can be modeled as an open circuit if the potential across the junction is less than the turn-on voltage, and a short circuit if the potential is greater than the turn-on voltage.

Once the diode conducts, a relatively constant potential will drop across it.
A simple example shows the relative ease a constant-voltage model has over an exponential model.

For an exponential model, iterative method is needed to solve for current, whereas constant-voltage model requires only algebra.

\[ V_X = I_X R_1 + V_D \]

\[ = I_X R_1 + V_T \ln \frac{I_X}{I_S} \]

For \( V_X = 3V \):

\[ I_X = 2.2mA \]

For \( V_X = 1V \):

\[ I_X = 0.2mA \]
Reverse Breakdown

- When a large reverse bias voltage is applied, an enormous current will flow across the diode pushing the device into breakdown.
Zener VS. Avalanche Breakdown

- Zener breakdown is a result of the large electric field inside the depletion region that breaks electrons or holes off of their covalent bonds.
- Avalanche breakdown is a result of electrons or holes colliding with the fixed-ions inside the depletion region. If these collisions were with great force, more electrons or holes would be freed from their covalent bonds.
Chapter 3   Diode Circuits

- 3.1 Ideal Diode
- 3.2 PN Junction as a Diode
- 3.3 Applications of Diodes
After we have studied in detail the physics of a diode, it is time to study how it behaves as a circuit element and its many applications.
An important application of diode is cell phone charger. Diode acts as the black box (after transformer) that passes only the positive half of the stepped-down sinusoid.
Diode’s Action in The Black Box (Ideal Diode)

- The diode behaves as a short circuit during the positive half cycle (voltage across it is greater than zero), and an open circuit during the negative half cycle (voltage across it is less than zero).
In an ideal diode, if the voltage across it is greater than zero, current will flow from anode to cathode. However, if voltage across it is less than zero, no current will flow.

It is analogous to a water pipe that allows water to flow in one direction (depending on water pressure), while blocking the other direction.
Diode in Series

- Diodes cannot be connected in series randomly. For the circuits above, only a) can conduct current from A to C.
IV Characteristics of an Ideal Diode

\[ R = 0 \Rightarrow I = \frac{V}{R} = \infty \]

\[ R = \infty \Rightarrow I = \frac{V}{R} = 0 \]

- If the voltage across anode and cathode is greater than zero, the resistance of an ideal diode is zero and current becomes infinite. However, if the voltage is less than zero, the resistance becomes infinite and current is zero.
Anti-Parallel Ideal Diodes

- If two diodes are connected in anti-parallel, it acts as a short for all voltages.
Diode-Resistor Combination

- The IV characteristics of a diode-resistor combination is zero for negative voltages and Ohm's law for positive voltages.
Diode Implementation of OR Gate

- The circuit above shows an example of diode-implemented OR gate.
- Vout can only be either Va or Vb, not both.
**Input/Output Characteristics**

- When $Vin$ is less than zero, the diode opens, so $Vout = Vin$.
- When $Vin$ is greater than zero, the diode shorts, so $Vout = 0$.
A rectifier is a device that passes positive-half cycle of a sinusoid and blocks the negative half-cycle.

When Vin is greater than 0, diode shorts, so Vout = Vin, however when Vin is less than 0, diode opens, no current will flow thru R1, Vout = IR1 = 0.
The averaged value of a rectifier output can be used as a signal strength indicator of the input, since $V_{out,avg}$ is proportional to $V_p$, the input signal’s amplitude.
Diode’s application: Limiter

- The purpose of a limiter is to force the output to remain below certain value.
- As can be seen in a), the addition of a 1V battery forces the diode to turn on after V1 has become greater than 1V.
An interesting case occurs when VB (battery) varies its value.

As it can be seen, rectification fails if VB was greater than the input’s amplitude.
So far we have studied the ideal model for diode. However, there are still the exponential model for diode, and the constant voltage model.
Input/Output Characteristics with Ideal and Constant-Voltage Models

- The circuit above shows the difference between the ideal and constant-voltage model; the two models yield two different break points of slope.
Again, when using a constant-voltage model, the voltage drop across the diode is no longer zero but $V_{d,\text{on}}$ when it conducts.
In this example, since Vin is connected to the cathode, the diode conducts when Vin is very negative.

The break point where the slope changes is when the current across R1 is equal to the current across R2.
In this example, since the two diodes have different cross-section area, only exponential model can be used.

The two currents are solved by summing them with $I_{in}$, and equate their voltages.

\[
I_{D1} = \frac{I_{in}}{1 + \frac{I_{s2}}{I_{s1}}}
\]

\[
I_{D2} = \frac{I_{in}}{1 + \frac{I_{s1}}{I_{s2}}}
\]
Another Constant-Voltage Model

This example shows the importance of good initial guess and careful post-confirmation.
The voltage drops across three diodes are used to charge cell phones.

However, if \( I_X \) changes, iterative method is often needed to obtain a solution, thus motivating a simpler technique.

\[
V_{out} = 3V_D = 3V_T \ln \frac{I_X}{I_s}
\]
Assuming the diode was initially biased at point A, and a small perturbation in voltage causes the diode’s bias to be at point B. If this perturbation in voltage were small, the resulting current perturbation became a linear function of voltage perturbation.

This technique is known as small-signal analysis.
If two points on the IV curve of a diode were close enough, the trajectory connecting the first to the second point is like a line, with the slope being the proportionality factor between change in voltage and change in current.

\[
\frac{\Delta I_D}{\Delta V_D} = \left. \frac{dI_D}{dV_D} \right|_{V_D=V_{D1}} = \frac{I_s}{V_T} \exp \frac{I_{D1}}{V_T} = \frac{I_{D1}}{V_T}
\]
Since there’s a linear relationship between the small signal current and voltage of a diode, the diode can be seen as a linear resistor when only small signal is interested.
Small Sinusoid Analysis

If a sinusoidal voltage with small amplitude is applied across the diode, the resulting current is also a small sinusoid perturbing around a DC value.

\[ V(t) = V_0 + V_p \cos \omega t \]
\[ I_D(t) = I_0 + I_p \cos \omega t \]
\[ = I_s \exp \left( \frac{V_0}{V_T} + \frac{V_T}{I_0} V_p \cos \omega t \right) \]
Regardless if the perturbation were current or voltage, small signal analysis applied in either case and the proportional constant was the inverse of each other.
With our understanding of small signal analysis, we can revisit our cell phone charger example and easily solve it with just algebra instead of iteration.

\[ v_{out} = \frac{3r_d}{R_1 + 3r_d} v_{ad} \]

\[ = 11.5 mV \]
In this example we study the effect of cell phone pulling some current from the diodes. Using small signal analysis, this is easily done. However, imagine the nightmare we would have to go thru, if we would to solve it using non-linear large signal technique.

\[ \Delta V_{out} = \Delta I_D \cdot (3r_d) \]
\[ = 0.5mA(3 \times 4.33\Omega) \]
\[ = 6.5mV \]
Applications of Diode

- Hereon after, we are going to go over several important applications of diode.
- The outline above gives us a direction.
As mentioned earlier, one of the most common application of diode is half-wave rectification, where either the positive or negative half of the input is cut off.
If the resistor in half-wave rectifier is replaced by a capacitor, a fixed voltage output is obtained since the capacitor (assumed ideal) has no path to discharge.
If the diode were assumed to be ideal, a) plots $V_{out}$, and b) plots the voltage difference between the diode.

Note that b) is just like $V_{in}$ only shifted down.
If there were a loading resistor across the capacitor, a path is available for capacitor to discharge. Therefore, $V_{out}$ will not be constant and a ripple exists.
If $C_1$ is varied, $V_{out}$ will be very different.

For large capacitance, $V_{out}$ will have small ripple. And for small capacitance, $V_{out}$ will have large ripple.
Peak to Peak amplitude of Ripple

\[ V_{out}(t) = (V_p - V_{D, on}) \exp \left( -\frac{t}{R_L C_1} \right) \quad 0 \leq t \leq T_{in} \]

\[ V_{out}(t) \approx (V_p - V_{D, on}) (1 - \frac{t}{R_L C_1}) \]

\[ \approx (V_p - V_{D, on}) - \frac{V_p - V_{D, on}}{R_L} \frac{t}{C_1} \]

\[ V_R \approx \frac{V_p - V_{D, on}}{R_L} \frac{T_{in}}{C_1} \]

\[ \approx \frac{V_p - V_{D, on}}{R_L C_1 f_{in}} \]

- Ripple voltage becomes a problem if it goes above 5 to 10% of the input voltage.
- The ripple’s amplitude is the decaying part of the exponential.
Maximum Diode Current

The diode has its maximum current is at $t_1$, since that’s when the slope of $C_1$ is greatest.

This current has to be carefully controlled so it does not damage the device.

\[
I_p \approx C_1 \omega_{in} V_p \sqrt{\frac{2V_R}{V_p}} + \frac{V_p}{R_L}
\approx \frac{V_p}{R_L} \left( R_L C_1 \omega_{in} \sqrt{\frac{2V_R}{V_p}} + 1 \right)
\]
A full-wave rectifier passes both the negative and positive half cycles of the input, only it inverts the negative half of the input.

As will be proved later, it reduces the ripple by a factor of two.
The figure above shows the evolution of full-wave rectifier.
Figure (e) and (f) show the topology that inverts the negative half cycle of the input.
The figure above shows a full-wave rectifier, where D1 and D2 passes/inverts the negative half cycle of input and D3 and D4 passes the positive half cycle.
Input/Output Characteristics of a Full-Wave Rectifier (Constant-Voltage Model)

- The figure above shows the input/output characteristics of a full-wave rectifier.
- Note the $2V_{D,\text{on}}$ dead-region around $V_{\text{in}} = 0$; as $V_{\text{in}}$ has to overcome two diode turn-on voltage.
Since C1 only gets $\frac{1}{2}$ of period to discharge, ripple voltage is decreased by 2 times. Also (b) shows that each diode is subjected to approximately one Vp reverse bias drop versus 2Vp in half-wave rectifier.
Current Carried by Each Diode in the Full-Wave Rectifier

The figure above shows the current carried by each diode during a full period.
Summary of Half and Full-Wave Rectifier

- The figure above summarizes the behavior of half-wave and full-wave rectifier.
- Full-wave rectifier is more suited for adapter and charger applications.
The ripple created by the rectifier can be unacceptable to sensitive load, therefore a regulator is required to obtain a very stable output.

The figure shows a very primitive combination of transformer, rectifier, and regulator.
Voltage regulation can be accomplished with Zener diode. Since it’s small-signal resistance is very small, large change in the input will not be reflected at the output.

\[ V_{out} = \frac{r_D}{r_D + R_1} V_{in} \]
Line Regulation VS. Load Regulation

Line regulation is the suppression of change in \( V_{out} \) due to change in \( V_{in} \) (b).

Load regulation is the suppression of change in \( V_{out} \) due to change in load current (c).

\[
\frac{V_{out}}{V_{in}} = \frac{r_{D1} + r_{D2}}{r_{D1} + r_{D2} + R_1}
\]

\[
\left| \frac{V_{out}}{I_L} \right| = (r_{D1} + r_{D2}) \parallel R_1
\]
The figure above shows an evolution of AC-DC converter, first a half-wave rectifier, then a full-wave rectifier, and finally a full-wave rectifier with regulator.
The motivation of having limiting circuits is to keep the signal below a threshold so it will not saturate the entire circuitry.

It is needed in a receiver when it is close to a base station, so the signal will not become too large.
Input/Output Characteristics

- Its input/output characteristics are shown above.
- Note the clipping of the output voltage.
Diode Implemented Limiting Circuit: Positive Cycle Clipping

- As was studied in the past, the combination of resistor-diode creates limiting effect.
- The above configuration creates positive cycle clipping.
Similarly, the above circuit provides limiting effect with negative cycle clipping.
Diode Implemented Limiting Circuits: Positive and Negative Cycle Clipping

- By combining the two topologies above (antiparallel diode connection), it is possible to obtain limiting effects in both positive and negative half cycles.
The Full-Blown Voltage Limiting Circuits

- Finally, by adding two batteries in series with the antiparalle diodes, we can control the limiting voltage.
Non-idealities in Limiting Circuits

- The clipping region is not exactly flat since as $V_{in}$ increases the currents that diodes carry will change, so will the turn-on voltage.
- During the passing region, the slope need not to be one. Amplification can make it greater than one.
Before we go further into our studying of diodes, we should first review our understanding of capacitors as they will play an important role in the circuits we will learn later.

The figure above presents two types of capacitive circuits and their output response to input change.
Waveform Shifter: Peak at -2Vp

- As Vin increases, D1 is turned on and Vout is zero.
- As Vin decreases, D1 is turned off, and Vout drops with Vin from zero. The lowest Vout can go is -2Vp.
Similarly, when the polarity of the diode is switched, a voltage doubler with peak value at 2Vp can be conceived.
The output will increase by $V_p$, $V_p/2$, $V_p/4$, etc in each input cycle, eventually settle to $2V_p$. 

Voltage Doubler
Current Thru D1 in the Voltage Doubler

- The figure above shows the current that D1 carries with respect to time. This current is the same current thru C1 when D1 conducts.
Since when a diode conducts and if its current is well-defined, the voltage drop across it is constant; therefore, it can be used as a voltage shifter. Figure (b) provides a practical example of it.
Here we have a voltage shifter, that shifts the original voltage by $2V_{D,\text{on}}$.
One of the most important electronic device is a switch. It finds application in logic circuits and data converters. The figure above illustrates a diode implemented switch.
Chapter 4  Physics of Bipolar Transistors

- 4.1 General Considerations
- 4.2 Structure of Bipolar Transistor
- 4.3 Operation of Bipolar Transistor in Active Mode
- 4.4 Bipolar Transistor Models
- 4.5 Operation of Bipolar Transistor in Saturation Mode
- 4.6 The PNP Transistor
For the circuit shown in part e) of the previous slide, a small feedthrough from input to output via the junction capacitors exists even if the diodes are reverse biased. Therefore, C1 has to be large to minimize this feedthrough.

\[
\Delta V_{out} = \frac{C_j / 2}{C_j / 2 + C_1} \Delta V_{in}
\]
In the chapter, we will study the physics of bipolar transistor and derive large and small signal models.
A voltage-dependent current source can act as an amplifier.

If $KR_L$ were greater than 1, then the signal amplifies.
Regardless of the internal resistance that voltage-dependent current source sees, the magnitude of amplification remains unchanged.

This point will prove useful in later analysis.
A three-terminal exponential voltage-dependent current source is shown above.

Ideally, bipolar transistor can be modeled as such.
Bipolar transistor can be thought as a sandwich of three doped Si regions. With the outer doping being the same, and the middle doping different. The figure above shows an example of npn configuration.

Emitter emits carriers and collector collects them while base control how many carriers are sent.
Sweeping of Carriers

- Reverse biased PN junction creates a large electric field that sweeps any external injected minority carriers to their majority region.
- This ability proves essential in the proper operation of a bipolar transistor.
When the base and emitter terminals are forward biased and base and collector terminals are reverse biased, the bipolar transistor is operating in forward active region.

We may be tempted to model the transistor as (b), however, we will later see that this is wrong.
If we studied the structure of bipolar transistor more carefully, we would see that it is more than just a back to back diodes from collector to emitter.

Collector also carries current due to carriers-sweeping from the built-in electric field.
Since base and emitter terminals are forward biased, its built-in potential is very low. Thus drift current is negligible in base, and diffusion current dominates. Since there’s a carrier gradient from the base/emitter junction and base/collector junction, the dominance of diffusion current is confirmed.
Collectors Current

\[ I_C = \frac{A_E q D_n n_i^2}{N_E W_B} \left( \exp \frac{V_{BE}}{V_T} - 1 \right) \]

\[ I_C = I_S \exp \frac{V_{BE}}{V_T} \]

\[ I_S = \frac{A_E q D_n n_i^2}{N_E W_B} \]

- Applying the law of diffusion, we can determine the charge flow across the base region into the collector.
- The equation above shows that the collector current is indeed a voltage-controlled element, thus a good candidate as an amplifier.
Parallel Combination of Transistors

When two transistors are put in parallel and experience the same potential across all three terminals, they can be thought of as a single transistor with twice the emitter area.
Although a transistor is a voltage to current converter, output voltage can be obtained by inserting a loading resistor at the output and allowing the controlled current to pass thru it.
Ideally speaking, the collector current does not depend on the collector to emitter voltage. This property allows the BJT to behave as a constant current source when its base to emitter voltage is fixed.
Base current consists of two components. 1) Reverse injection of holes into the emitter and 2) recombination of holes with electrons coming from the emitter.

Base current is often expressed as a fraction of collector current, with beta being the current gain.

\[
I_C = \beta I_B
\]
Applying Kirchoff’s current law to the transistor, we can easily find the emitter current.

\[ I_E = I_C + I_B \]

\[ I_E = I_C \left( 1 + \frac{1}{\beta} \right) \]
Summary of Currents

\[ I_C = I_s \exp \frac{V_{BE}}{V_T} \]

\[ I_B = \frac{1}{\beta} I_s \exp \frac{V_{BE}}{V_T} \]

\[ I_E = \frac{\beta + 1}{\beta} I_s \exp \frac{V_{BE}}{V_T} \]

\[ \frac{\beta}{\beta + 1} = \alpha \]
Just like the diodes, a large signal model for BJT can also be derived.

Since base-emitter junctions are forward biased, a diode is placed there.

A voltage controlled current source is placed between the collector and emitter terminals.
As $R_L$ increases, $V_x$ will drop and eventually forward bias the collector and base junctions. This will force the BJT out of active region.

Therefore, there exists a maximum collector resistance.
The IV characteristics above show the exponential relationship of the collector current and its independence from collector and emitter potential difference.
IV Characteristics of Base Current

- Since base current and collector current are related by a beta factor, similar IV characteristics can be derived.
Transconductance

A measure of how good the BJT’s voltage to current conversion capability is its transconductance or \( g_m \).

It will later be shown that \( g_m \) is one of the most important parameters in analog design.

\[
g_m = \frac{d}{dV_{BE}} \left( I_S \exp \frac{V_{BE}}{V_T} \right)
\]

\[
g_m = \frac{1}{V_T} I_S \exp \frac{V_{BE}}{V_T}
\]

\[
g_m = \frac{I_C}{V_T}
\]
Visualization of Transconductance

- $g_m$ can be visualize (indeed is) as the slope of $I_C$ versus $V_{BE}$.
- A large $I_C$ has a large slope and therefore a large $g_m$. 
Transconductance and Area

- When the area of a transistor is increased by $n$, $I_s$ will increase by $n$ and therefore $g_m$ will increase accordingly by $n$. 
Transconductance and $I_c$

- The figure above shows for a fixed voltage swing, the current excursion around $I_{C2}$ is larger than it would be around $I_{C1}$. This is because $g_m$ at $I_{C2}$ is larger than that of $I_{C1}$'s.
Small signal model is derived by perturbing every two terminals while fixing the third terminal constant and analyzing the change in current of all three terminals. We then represent these changes with controlled sources or resistors.
By varying the potential difference between base and emitter, we discover that there’s a controlled current flowing from collector and a small signal resistor between base and emitter.
Since $V_{CE}$ has no effect on the collector current, it will not contribute to the small signal model, thus our previous model still holds.

It can be shown that $V_{CB}$ has no effect on the small signal model as well.
Here, small signal parameters are calculated from DC operating point and are used to calculate the change in collector current due to a change in $V_{BE}$.
In this example, a resistor is placed between the power supply and collector, therefore, providing an output voltage.

This example sheds light on the amplification capability of BJT.
AC Ground

- Since the power supply remains constant with respect to the change in small signal, in analysis, it is regarded as a ground.

- In fact, any constant source can be regarded either as a ground for voltage or open for current in small signal analysis.
The claim that collector current does not depend on $V_{CE}$ is not accurate.

As $V_{CE}$ increases, the depletion region between base and collector increases, therefore the effective base width decreases, which leads to an increase in the collector current.
Early Effect Illustration

- With Early effect, collector current becomes larger than usual and a function of $V_{CE}$. 
Early effect can be realized in BJT as a non-ideal current source that varies with the potential drop across its terminals.
Early effect can be accounted in large signal model by simply changing the collector current with the corrected form.

- Base current does not change with Early effect.
Early effect and Small Signal Model

- Early effect can be accounted for in small signal model as an addition of an output resistor between collector and emitter.

\[
\begin{align*}
\Delta I_C &= \Delta V_{CE} \\
\Delta V &= r_o \Delta I_C \\
\end{align*}
\]

\[
\begin{align*}
r_o &= \frac{\Delta V_{CE}}{\Delta I_C} \\
r_o &= \frac{V_A}{I_s \exp \left(\frac{V_{BE}}{V_T}\right)} \\
r_o &\approx \frac{V_A}{I_C}
\end{align*}
\]
Summary of Ideas
BJT in Saturation

- When collector voltage drops below base voltage and forward biases the collector and base PN junction, base current will increase and therefore decrease the current gain factor, beta.
Since the PN junction between base and collector is forward bias, a diode is placed there as a reminder that effectively collector current drops. This is because part of the controlled current source’s current comes from the diode instead of the collector lead.
IV Characteristics in Saturation

- As BJT enters saturation, it seems a drastically drop in collector current.
- The speed of the BJT also drops in saturation.
Acceptable Range of $V_{CC}$ and $R_C$

\[ V_{CC} \geq I_C R_C + (V_{BE} - 400mV) \]

- In order to keep BJT at least in soft saturation region, the collector voltage must not fall below the base voltage by more than 400mV.
- A linear relationship can be derived for $V_{CC}$ and $R_C$ and an acceptable region can be chosen.
In deep saturation region, the transistor loses its voltage-controlled current capability and its collector and emitter voltage becomes constant.
With the polarities of emitter, collector, and base reversed, a PNP BJT is conceived.

All the principles that applied to NPN's also apply to PNP’s, with the exception that emitter is at a higher potential than base and base at a higher potential than collector.
A Comparison between NPN and PNP BJT’s

- For NPN BJT, collector current adds with base current to form emitter current; whereas for PNP BJT, emitter current loses base current to yield collector current.

- In order to operate in the active region, NPN BJT cannot allow collector voltage to be lower than base’s; whereas PNP BJT cannot allow collector voltage to be higher than base’s.
PNP Equations

\[ I_C = I_S \exp \frac{V_{EB}}{V_T} \]

\[ I_B = \frac{I_S}{\beta} \exp \frac{V_{EB}}{V_T} \]

\[ I_E = \frac{\beta + 1}{\beta} I_S \exp \frac{V_{EB}}{V_T} \]

\[ I_C = \left( I_S \exp \frac{V_{EB}}{V_T} \right) \left( 1 + \frac{V_{EC}}{V_A} \right) \text{ Early Effect} \]

- All the equations used for NPN BJT’s also apply here for PNP BJT’s with only the polarities of voltages switched.
As we can see, the only differences between large signal model for PNP BJT and that of NPN BJT are the direction of current flow and switched polarity of voltage in the current equation.
The figure above shows a simple case of PNP BJT biasing. Note that the emitter is at a higher potential than both the base and collector.
Similarly, PNP BJT’s can also perform signal amplification. Therefore, a set of small signal model and parameters are needed for PNP BJT’s.
The small signal model for PNP BJT is shown above, which is exactly IDENTICAL to that of NPN. This is not a mistake because the current direction is taken care of by the polarity of $V_{BE}$. 
Here we have a simple NPN BJT and its small signal model.
Here we have a PNP BJT configured in the same way as the previous NPN BJT, therefore their final small signal models are identical.
Here we have another example of PNP BJT, with the signal’s ground connected to the power supply which also connects to the emitter.

Since during small signal analysis, a constant voltage supply is considered to be AC ground, its final small signal model is identical to the previous two.
A more complicated small signal model with both NPN and PNP BJT’s are considered above.