Making Basic Measurements

The figures on the following pages show how to make basic measurements.

Warning

To avoid electric shock, injury, or damage to the Meter, disconnect circuit power and discharge all high-voltage capacitors before testing resistance, continuity, diodes, or capacitance.

Measuring Resistance

Measuring AC and DC Voltage

<table>
<thead>
<tr>
<th>Volts AC</th>
<th>Volts DC</th>
<th>Millivolts DC</th>
</tr>
</thead>
<tbody>
<tr>
<td>120.1 V</td>
<td>72.34 V</td>
<td>20.4 V</td>
</tr>
</tbody>
</table>

Measuring Capacitance

Testing for Continuity

Testing Diodes

Good Diode

Forward Bias

Single Beep

Reverse Bias

Bad Diode

Open

Measuring Temperature (Model 179 Only)

80BK1 Type K Thermocouple Probe

Vent or Pipe

272°F
### Specifications

<table>
<thead>
<tr>
<th>Function</th>
<th>Range ¹</th>
<th>Resolution</th>
<th>Accuracy ± ([% of Reading] + [Counts])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Model 175</td>
</tr>
<tr>
<td>AC mV</td>
<td>600.0 mV</td>
<td>0.1 mV</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td>DC mV</td>
<td>600.0 mV</td>
<td>0.1 mV</td>
<td>0.15 % + 2</td>
</tr>
<tr>
<td>DC Volts</td>
<td>60.00 V</td>
<td>0.001 V</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td>Continuity</td>
<td>600 Ω</td>
<td>1 Ω</td>
<td>Meter beeps at &lt; 25 Ω, beeper turns off at &gt; 250 Ω; detects opens or shorts of 250 μs or longer.</td>
</tr>
<tr>
<td>Ohms</td>
<td>600.0 kΩ</td>
<td>0.1 kΩ</td>
<td>0.9 % + 2</td>
</tr>
<tr>
<td>Capacitance</td>
<td>1000 nF</td>
<td>1 nF</td>
<td>1.2 % + 2</td>
</tr>
<tr>
<td>AC Amps</td>
<td>60.00 mA</td>
<td>0.01 mA</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td></td>
<td>400.0 mA</td>
<td>0.1 mA</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td></td>
<td>60.00 A</td>
<td>0.001 A</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td>Diode test</td>
<td>2.400 V</td>
<td>0.001 V</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td>Temperature</td>
<td>-40 °C to +400 °C</td>
<td>0.1 °C</td>
<td>1.0 % + 3</td>
</tr>
</tbody>
</table>

1. All AC voltage and AC current ranges are specified from 5 % of range to 100 % of range.
2. The 600 AC mV range can only be entered in the manual range mode.
3. Crest factor of ≤ 3 at full scale up to 500 V, decreasing linearly to crest factor ≤ 1.5 at 1000 V.
4. In the 9999 μF range for measurements to 1000 μF, the measurement accuracy is 1.2 % for all models.

### Models 175, 177 & 179

### Users Manual

<table>
<thead>
<tr>
<th>Function</th>
<th>Range ¹</th>
<th>Resolution</th>
<th>Accuracy ± ([% of Reading] + [Counts])</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Model 175</td>
</tr>
<tr>
<td>DC Amps</td>
<td>60.00 mA</td>
<td>0.01 mA</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td></td>
<td>400.0 mA</td>
<td>0.1 mA</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td></td>
<td>60.00 A</td>
<td>0.001 A</td>
<td>1.0 % + 3</td>
</tr>
<tr>
<td>Hz (AC- or DC-coupled, V or A input)</td>
<td>99.99 Hz</td>
<td>0.01 Hz</td>
<td>0.1 % + 1</td>
</tr>
<tr>
<td></td>
<td>99.99 kHz</td>
<td>0.001 kHz</td>
<td>0.1 % + 1</td>
</tr>
<tr>
<td>Temperature</td>
<td>-40 °C to +75 °C</td>
<td>0.1 °F</td>
<td>NA</td>
</tr>
</tbody>
</table>

MIN MAX AVG: Accuracy is the specified accuracy of the measurement function ± 12 digits for changes >200 ms in duration (± 40 digits in AC). Typical response time: 100 ms to 80 % of signal.

1. All AC voltage and AC current ranges are specified from 5 % of range to 100 % of range.
2. In mA and A ranges, frequency measurement is specified to 30 kHz.
SI UNITS*

The International System of Units (Système International d'Unités) is the name given in 1960 by the Conférence Générale des Poids et Mesures to the coherent system of units that has the following base units and quantities:

<table>
<thead>
<tr>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>meter</td>
<td>length</td>
</tr>
<tr>
<td>kilogram</td>
<td>mass</td>
</tr>
<tr>
<td>second</td>
<td>time</td>
</tr>
<tr>
<td>ampere</td>
<td>electric current</td>
</tr>
<tr>
<td>kelvin</td>
<td>thermodynamic temperature</td>
</tr>
<tr>
<td>mole</td>
<td>amount of substance</td>
</tr>
<tr>
<td>candela</td>
<td>luminous intensity</td>
</tr>
</tbody>
</table>

Units of this system are called SI units.

Prefixes

The prefixes used to indicate multiples or submultiples of units are listed in Table 1.

Symbols for prefixes are printed in roman type, without space between the prefix and the symbol for the unit. The distinctions between upper- and lower-case letters must be observed.

Compound prefixes should not be used:

<table>
<thead>
<tr>
<th>Use</th>
<th>Do not use</th>
</tr>
</thead>
<tbody>
<tr>
<td>tera</td>
<td>T</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
</tr>
</tbody>
</table>

When a symbol representing a unit that has a prefix carries an exponent, this indicates that the multiple (or submultiple) unit is raised to the power expressed by the exponent. For example:

2 cm$^3$ = 2(cm)$^3$ = $2 \times 10^{-3}$ m$^3$

1 ms$^{-1}$ = 1(ms)$^{-1}$ = $1 \times 10^{-3}$ s$^{-1}$ = 10$^3$s$^{-1}$

Symbols for Units†

Unit symbols are letters, combinations of letters, or other characters that may be used in place of the names of the units.

Example: In the expression $I = 150$ mA, $I$ is the symbol for a physical quantity (current). $A$ is the symbol for the unit of current (ampere), and $m$ is the symbol for the prefix milli. Together, $m$ and $A$ form the symbol for a submultiple unit of current, the milliampere.

When an unfamiliar unit symbol is first used in text, it should be followed by its name in parentheses. Only the symbol need be used thereafter.

Symbols for units are written in lower-case (small) letters, except for the first letter if the name of the unit is derived from a proper name, and except for a very few that are not formed from letters. Every effort should be made to follow the distinction between upper- and lower-case letters, even if the symbols appear in applications where the other lettering is in upper-case style.

Symbols for units are printed in roman (upright) type. Their form is the same for both singular and plural, and they are not followed by a period. When there is risk of confusion in using the standard symbols, e.g., "$l$" for liter and "$s$" for second, the name of the unit should be spelled out.

When a compound unit is formed by multiplication of two or more units, its symbol consists of the symbols for the separate units joined by a raised dot (for example, N m for newton meter). The dot may be omitted in the case of familiar compounds if no confusion would result. For example, V · s and V s are both acceptable representations of the unit weber. Wb.

When a unit symbol prefix is identical to a unit symbol, special care must be taken. For example, the symbol $m$ · N indicates the product of the units meter and newton, while $m$N is the symbol for millinewton. Hyphens should not be used in symbols for compound units. Positive and negative exponents may be used within
# Table 2. Preferred Values*

<table>
<thead>
<tr>
<th>Name of Series</th>
<th>USA Standard Z17.1-1973†</th>
<th>USA Standard C83.2-1971 (R 1977)‡</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>†'5''</td>
<td>± 20% (E6)</td>
</tr>
<tr>
<td>Percent step size</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>Step multiplier</td>
<td>(10)²⁻³ = 1.58</td>
<td>(10)²⁻¹ = 1.26</td>
</tr>
<tr>
<td>Values in the series</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use decimal multipliers for smaller or larger values</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>12.5</td>
<td>12.5</td>
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<tr>
<td>15</td>
<td>15</td>
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<td>82</td>
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</tr>
<tr>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

* ANSI Standard C83.2-1971 applies to most electronics components. It is the same as EIA Standard RS-385 (formerly GEN-102) and agrees with IEC Publication 63. ANSI Standard Z17.1-1973 covers preferred numbers and agrees with ISO 3 and ISO 497.
† '20'' series with 12-percent steps ((10)²⁻¹² = 1.122 multiplier) and a '40'' series with 6-percent steps ((10)¹⁻⁴ = 1.059 multiplier) are also standard.
‡ Associate the tolerance ±20%, ±10%, or ±5% only with the values listed in the corresponding column. Thus, 1300 ohms may be either ±10 or ±5, but not ±20 percent; 750 ohms may be ±5, but neither ±20 nor ±10 percent.

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## Constant-Humidity Tests

40 °C, 90 to 95% RH; 4, 10, 21, or 56 days.
66 °C, = 100% RH; 48, 96, or 240 hours (primarily for small items).

## Cycling Humidity Tests

Fig. 1 shows a number of cycling humidity tests. (See applicable chart in standard for full details.) Preconditioning is customary before running cycle series. RH = relative humidity.

## High-Altitude Tests

Information regarding high-altitude tests is given in Table 4.

## Vibration Tests

The purposes of vibration tests are:
(A) Search for resonance.
(B) Determination of endurance (life) at resonance (at specific frequencies).
(C) Determination of deterioration resulting from
Example 6-5

If a periodic function displayed on the scope screen has a distance of 4 cm between the beginning and end of a cycle, and if the Time/div. control is set to 1 ms/div, what is the frequency of the waveform?

Solution. First find the time duration of one waveform:

\[ t = \text{horizontal distance} \times \text{horizontal sweep setting} \]
\[ = 4 \text{ div} \times 0.001 \text{ s/\text{div}} \]
\[ = 0.004 \text{ s} \]

Since in this case \( t = T \), use

\[ f = \frac{1}{T} = \frac{1}{0.004} = 250 \text{ Hz} \]

then if \( V_p = 0.2 \text{ volts} \)

\[ V = V_p \sin \omega t = 0.2 \sin (2\pi f t) = 0.2 \sin (1570t) \]

Phase Measurements (Triggered-Sweep Method)

The phase difference between two waveforms of the same frequency can be found by using the triggered-sweep method and the Lissajous figures method. In this section the triggered-sweep method is discussed.

The triggered-sweep method for determining phase difference compares the phase of two signals by using one signal as the reference. The shift in the position of the second signal compared to the reference signal can be used to calculate the phase difference between the signals.

To make the measurement, the phase of one signal is chosen as zero and the scope display is calibrated to indicate this choice. The calibration procedure involves setting the scope to External Trigger, the Level to zero, and the Slope to plus so that the sweep triggers when a trigger signal crosses zero with a plus slope. The first signal, \( A \), is connected then to both the vertical inputs and the external trigger terminals. The waveform displayed by the scope is like the waveform shown in Fig. 6-31. Next, the vertical input signal is changed from signal \( A \) to signal \( B \). Signal \( A \)

![Figure 6-31 Phase-difference measurement using the triggered-sweep method.](image)
How to Operate an Oscilloscope

![Diagram](image)

Figure 6-32 How to determine the phase angle from the triggered sweep display:
(a) calibration of horizontal axis so that 180° equals nine divisions; (b) phase of signal B is equal to −θ for this type of position shift; (c) phase of signal B is θ = 180° − θ for this type of position shift.

remains connected as the external triggering signal. Thus, if signal A triggers a sweep when signal B is not at the same level and slope, the display of signal B will be shifted in position along the horizontal (time) axis. To calibrate the time axis so that it corresponds to 20°/div., use the Variable Sweep Time control to adjust the display of the waveform so that half a cycle of signal A corresponds to nine divisions [Fig. 6-32(a)]. Then the phase shift can be found by measuring the distance to the first zero-crossing of signal B [Fig. 6-32(b) and (c)].

Example 6-6

Let $V_p = 3.4$ volts and $\omega = 23$ radians/second for signal B in Fig 6-31. If $\theta = 0.78$, write the equation for signal B if signal A is used as the reference.

$$V = V_p \sin (\omega t + \theta) = 3.4 \sin (23t - 0.78)$$

Note: $\theta$ is lagging (or minus) since it arrives later in time.

Lissajous Figures

If two sine waves are simultaneously fed to an oscilloscope (one to the vertical inputs and the other to the horizontal inputs) and the scope is set to operate in the X-Y mode, the resulting display on the scope screen is referred to as a Lissajous pattern. If the two sine waves are of the same frequency and phase, the Lissajous pattern will be a diagonal line. If the sine waves are of the same frequency but out of phase by 90°, the pattern will be an ellipse (if the amplitudes are also equal, the ellipse will instead be a circle). Figure 6-33 shows how Lissajous patterns result from the input of two sine waves.

The numbered dots on these figures trace the position of the electron beam as time and the magnitudes of the applied sine waves change. If the two signals are not of equal frequencies, the pattern will not be a diagonal, ellipse, or circle, but it will be some other unusual gyrating pattern. Thus, if the frequency of one signal is known, the other can be found by varying the known frequency source until a steady Lissajous pattern is displayed.
Figure 6-33 How Lissajous patterns are generated: (a) sine waves of equal frequency and phase applied to both horizontal and vertical plates; (b) sine waves of equal frequency and amplitude but with a phase difference of 90° applied to both the horizontal and vertical plates.

In addition to Lissajous patterns for measuring frequency, there other methods such as the modulated ring pattern, broken ring pattern, and broken line pattern. All of these are obtained by using a procedure very similar to the Lissajous pattern method and equipment interconnection. None of these methods, however, are used in modern laboratories where a high degree of accuracy and speed is required. Instead, digital frequency counters and phase meters are used. A 9-digit meter can resolve a period, frequency, or phase with a resolution of 1 LSD. Determining frequency or phase shift is only one application of the x-y mode. For any time two variables that are interdependent but not time dependent, the x-y setting will produce a display of the relationship. Some instrumentation examples would be the simultaneous display of the pressure and volume of a liquid, the speed and torque of a motor, and the deflection and force on a structural beam.
How to Operate an Oscilloscope

Frequency Measurements Using the X-Y Mode

Since the sweep times of the sweep waveform are usually calibrated to within 5 percent of their stated values, frequency measurements using the triggered-sweep method can be in error up to this amount. However, if an accurate, adjustable frequency source is applied to the horizontal input of a scope, an unknown frequency can be determined much more accurately by comparison (Fig. 6-34). This is done by varying the frequency of the accurate frequency source until either a Lissajous pattern of a circle or an ellipse appears on the screen. The appearance of the steady Lissajous pattern indicates that the frequencies of both applied signals are equal.

If it is not possible to adjust the frequency of the source to obtain a circle or ellipse, the known frequency should be adjusted until a stationary Lissajous pattern with a number of loops is reached. The ratio of the number of horizontal to vertical loops of the stationary pattern yields the unknown frequency (Fig. 6-35).

Phase Measurement Using Lissajous Patterns

To measure phase difference between two sine waves, they must by definition be of the same frequency. (The phase difference between two sine waves of different frequencies is meaningless.) Therefore, if two equal-frequency sine waves are fed to the vertical and horizontal inputs, respectively, the display on the scope screen will be a stable Lissajous pattern. The characteristics of the shape of the pattern allow the phase difference between the two signals to be determined. If the equations of the two waves are

\[ X = C \sin \omega t \]  \hspace{1cm} (6-8)

and

\[ Y = B \sin (\omega t + \theta) \]  \hspace{1cm} (6-9)

the phase difference \( \theta \) is found from the Lissajous pattern by the equation

\[ \frac{A}{B} = \sin \theta \]  \hspace{1cm} (6-10)

where \( A \) is the point where the ellipse crosses the \( Y \) axis (Fig. 6-36).

![Figure 6-34](image) Connections for measuring an unknown frequency by comparison to a known frequency.
Voltage Versus Current Display of Two-Terminal Devices

The determination of voltage versus current characteristics ($V$ versus $I$) of two- and three-terminal devices is usually a preliminary step towards the useful application of the devices as circuit elements. In the case of nonlinear devices, such as diodes and transistors, a graphical display of the $V$–$I$ characteristic is usually the most efficient means of displaying the $V$ versus $I$ data related to device operation. In this section we see how the oscilloscope can be used to display the $V$–$I$ characteristics of two-terminal devices by using semiconductor diodes as the demonstration vehicle.

In a later section of this chapter dealing with curve tracers, we shall see how the $V$–$I$ characteristics of three-terminal devices (such as bipolar and FET transistors) can be displayed.

Actual diodes are two terminal devices that have nonlinear $V$–$I$ characteristics. The current, $I_D$, flowing in semiconductor diodes is approximately found from the equation
The constant, $I_o$, is the reverse-saturation current of the diode (typically very small, $\approx 10^{-12} \text{A}$); $V$ is the voltage applied across the diode; $q$ is the electronic charge, $1.6 \times 10^{-19} \text{C}$; $k$ is Boltzmann's constant, $1.38 \times 10^{-23} \text{J} \cdot \text{K}^{-1}$; and $T$ is the temperature in °K. Therefore, the quantity $q/kT$ is equal to 0.026 V at room temperature ($T = 300°\text{K} = 26°\text{C}$). The graphical form of the voltage versus current characteristic of semiconductor diodes as described by Eq. (6-11) is shown in Fig. 6-37(a). In this figure we see that when the voltage across the diode, $V$, is positive (forward-bias condition), and is several times the value of $q/kT$ (i.e., $V \gg q/kT$), the current increases rapidly with increasing voltage. When the applied voltage is negative (reverse-bias condition) Eq. (6-11) predicts that $I_D \approx -I_o$. Therefore, the reverse current of the diode is constant and independent of the applied reverse-bias voltage. Most commercially available diodes exhibit an essentially constant value of $I_o$ for negative values of $V$. However, some diodes possess a pronounced (and possible unacceptable) increase in reverse current with increasing reverse voltage. In addition, at some value of reverse-bias voltage, real diodes exhibit an abrupt departure from Eq. (6-11). At this critical voltage (called the reverse-breakdown voltage), a large reverse current flows and the diode is said to be operating in the breakdown region.

An oscilloscope can be used to graphically display the $V$–$I$ relationship of the diode. The circuit shown in Fig. 6-37 demonstrates how this can be done. (Note that the same circuit can also be used to display the $V$–$I$ characteristics of virtually all other two terminal devices as well.) The oscilloscope (one with a single-ended input is shown in Fig. 6-37) is used in its $X$–$Y$ mode of operation. The sine-wave oscillator applies a sinusoidal voltage (60Hz is a convenient frequency) across the diode. The voltage appearing across the diode is also applied to the horizontal input of the scope. The current through the diode, $I_D$, is displayed as a vertical deflection, since $I_D$ is proportional to the voltage appearing across the current sampling resistor, $R_1$, of the circuit. If $R_1 = 1 \text{kΩ}$, as in Fig. 6-37, the vertical voltage sensitivity of the scope display (in V/div) is automatically converted to mA/div. The resistor, $R_1$, also performs the function of limiting the maximum power dissipation in the diode.
Supplement to Ex. 8

Substituting in (6.9), we obtain
\[ \delta(t)Cv(t+0) - u(t)v(t+0)Ge^{-tRC} + Gu(t)v(t+0)e^{-tRC} = \delta(t) \]
After cancellation, the only term that remains on the left-hand side is
\[ Cv(t+0) \delta(t) \]; since it must balance the term \( \delta(t) \) in the right-hand side, we obtain \( \gamma(t+0)C = 1 \) equivalently.
\[ \gamma(t+0) = \frac{1}{C} \]
Inserting this value of \( \gamma(t+0) \) into (6.12), we conclude that the solution of (6.9) is actually \( h \), the impulse response calculated previously.

Remark
We have just shown that the solution of the differential equation
\[ C \frac{d}{dt} v(t) + Gu = \delta \quad \text{with} \quad v(t-0) = 0 \]
for \( t \geq 0 \) is identical with the solution of
\[ C \frac{d}{dt} v(t) + Gu = 0 \quad \text{with} \quad v(t+0) = \frac{1}{C} \]
for \( t \geq 0 \). This can be seen by integrating both sides of (6.9) from \( t = 0^- \) to \( t = 0^+ \) to obtain
\[ Cr(t+0) - Cv(t-0) + G \int_{0^-}^{t} r(t')dt' = 1 \]
Since \( v \) is finite, \( G \int_{0^-}^{t} r(t')dt' = 0 \), and since \( v(t-0) = 0 \), we obtain
\[ v(t+0) = \frac{1}{C} \]
In Eq. (6.13) the effect of the impulse at \( t = 0 \) has been taken care of by the initial condition at \( t = 0^+ \).

7 Step and Impulse Responses for Simple Circuits

Example 1
Let us calculate the impulse response and the step response of the RL circuit shown in Fig. 7.1. The series connection of the linear time-invariant resistor and inductor is driven by a voltage source. As far as the impulse response is concerned, the differential equation for the current \( i \) is
\[ L \frac{di}{dt} + Ri = \delta \quad i(t-0) = 0 \]
If we confine our attention to the values of \( t > 0 \), this problem is equiva-
Fig. 7.1 (a) Linear time-invariant RL circuit: \( v \) is the input and \( i \) is the response; (b) impulse response; (c) step response.

The solution is

\[
(7.3) \quad i(t) = h(t) = \frac{1}{L} u(t) e^{-\frac{t}{L/R}}
\]

The step response can be obtained either from integration of (7.3) or directly from the differential equation

\[
(7.4) \quad \dot{i}(t) = \frac{1}{R} u(t)(1 - e^{-\frac{t}{L/R}})
\]

The physical explanation of the step response of the series RL circuit is now given. As the step of voltage is applied to the circuit, that is, at \( 0^+ \), the current in the circuit remains zero because, as we noted earlier, the current through an inductor cannot change instantaneously unless there is an infinitely large voltage across it. Since the current is zero, the voltage across the resistor must be zero. Therefore, at \( 0^+ \) all the voltage of the voltage source appears across the inductor; in fact \( \frac{di}{dt}_{0^+} = 1/L \). As time increases, the current increases monotonically, and after a very long time,
the current becomes practically constant. Thus, for large $t$, $di/dt \approx 0$; that is, the voltage across the inductor is zero, and all the voltage of the source is across the resistor. Therefore, the current is approximately $1/R$. In the limit we reach what is called the steady state and $i = 1/R$. We conclude that the inductor behaves as a short circuit in the steady state for a step voltage input.

Example 2 Consider the circuit in Fig. 7.2, where the series connection of a linear time-invariant resistor $R$ and a capacitor $C$ is driven by a voltage source. The current through the resistor is the response of interest, and the problem is to find the impulse and step responses. The equation for the current $i$ is given by writing KVL for the loop; thus

$$\frac{1}{C} \int_0^t i(t') \, dt' + Ri(t) = u_s(t)$$

Let us use the charge on the capacitor as the variable; then (7.5) becomes

$$\frac{q}{C} + R \frac{dq}{dt} = u_s(t)$$

Impulse response:

with the initial

Condition of (7.3) or

The $RL$ circuit is then, that is, at $t \to \infty$ as noted earlier, the voltage, unless there is zero, the voltage of the $1/L$. As time a very long time,

Fig. 7.2 (a) Linear time-invariant $RC$ circuit: $u_s$ is the input and $i$ is the response; (b) step response; (c) impulse response.
Since we have to find the step and impulse responses, the initial condition is \( q(0^-) = 0 \). If \( u_i \) is a unit step, (7.6) gives

\[
q_i(t) = u(t)C(1 - e^{-t/RC})
\]

and by differentiation, the step response for the current is

\[
i_i(t) = j_i(t) = \frac{1}{R} u(t)e^{-t/RC}
\]

If \( u_i \) is a unit impulse, (7.6) gives

\[
q_i(t) = \frac{1}{R} u(t)e^{-t/RC}
\]

and, by differentiation, the impulse response for the current is

\[
i_i(t) = h(t) = \frac{1}{R} \delta(t) - \frac{1}{R^2 C} u(t)e^{-t/RC}
\]

We observe that in response to a step, the current is discontinuous at \( t = 0 \); \( i_i(0^+) = 1/RC \) as we expect, since at \( t = 0 \) there is no charge (hence no voltage) on the capacitor. In response to an impulse, the current includes an impulse of value \( 1/RC \), and, for \( t > 0 \), the capacitor discharges through the resistor.

The step and impulse responses for simple first-order linear time-invariant circuits are tabulated in Table 4.1.

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**Time-varying Circuits and Nonlinear Circuits**

Up to this point we have analyzed almost exclusively linear time-invariant circuits. We have studied the implications of the linearity and of the time invariance of element characteristics as far as the relation between input and output is concerned. In this section we shall first summarize the main implications of linearity and of time invariance of element characteristics. Next we shall consider examples of circuits with nonlinear and of time-varying elements to demonstrate that without linearity and time invariance these main implications are no longer true.

In our study of first-order circuits we have seen that if the circuits are linear (time-invariant or time-varying), then

1. The zero-input response is a linear function of the initial state.
2. The zero-state response is a linear function of the input.
3. The complete response is the sum of the zero-input response and of the zero-state response.

We have also seen that if the circuit is linear and time-invariant, then

\[
\mathcal{L}[z(t)] = \mathcal{L}[z_i(t)]
\]

\( \tau \geq 0 \)
Table 4.1  Step and Impulse Responses for Simple Linear Time-invariant Circuits

<table>
<thead>
<tr>
<th>r(t;input)</th>
<th>v(t;response)</th>
</tr>
</thead>
</table>
|  \begin{array}{c}
    \text{C} \\
    \frac{d}{dt}v + \frac{1}{R}v = i_s \\
    \frac{1}{L} \phi + \frac{1}{L} \phi = i_s \\
    v = RT \frac{d}{dt}i_s \\
    v = \text{RTs} + \frac{1}{L} \frac{d}{dt}i_s \\
    v = \text{RTs} + \frac{1}{C} \int i_s(t) \, dt
    \end{array} |

<table>
<thead>
<tr>
<th>\delta(t)</th>
</tr>
</thead>
</table>
| \begin{array}{c}
    \text{R} \left( 1 - e^{-t/RC} \right) a(t) \\
    \frac{1}{C} e^{-t/RC} a(t) \\
    \frac{1}{C} e^{-t/RC} a(t) \\
    \frac{1}{C} e^{-t/RC} a(t) \\
    \frac{1}{C} e^{-t/RC} a(t) \\
    \frac{1}{C} e^{-t/RC} a(t)
    \end{array} |

<table>
<thead>
<tr>
<th>\delta(t)</th>
</tr>
</thead>
</table>
| \begin{array}{c}
    T = RC \\
    0.377 \frac{1}{C} e^{-t/RC} a(t) \\
    0.377 \frac{1}{C} e^{-t/RC} a(t) \\
    0.377 \frac{1}{C} e^{-t/RC} a(t) \\
    0.377 \frac{1}{C} e^{-t/RC} a(t) \\
    0.377 \frac{1}{C} e^{-t/RC} a(t)
    \end{array} |
Table 4.1  Step and Impulse Responses for Simple Linear Time-invariant Circuits (Continued)

<table>
<thead>
<tr>
<th>Input (e(t))</th>
<th>Response (v(t))</th>
<th>δ(t)</th>
<th>h(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(s)</td>
<td>v(s)</td>
<td>δ(t)</td>
<td>h(t)</td>
</tr>
<tr>
<td>L $\frac{di}{dt} + R(t) = e(t)$</td>
<td>$v(t)$</td>
<td>$\frac{1}{R} i(t) = (1 - e^{-t/RC}) u(t)$</td>
<td>$h(t)$</td>
</tr>
<tr>
<td>$\frac{R}{C} q(t) + \frac{1}{C} q(t) = e(t)$</td>
<td>$v(t)$</td>
<td>$\frac{1}{RC} e^{-t/RC} u(t)$</td>
<td>$h(t)$</td>
</tr>
<tr>
<td>$i = C \frac{de}{dt} + \frac{1}{R} e(t)$</td>
<td>$v(t)$</td>
<td>$\frac{1}{RC} e^{-t/RC} u(t)$</td>
<td>$h(t)$</td>
</tr>
<tr>
<td>$e(t)$</td>
<td>$v(t)$</td>
<td>$\frac{1}{R} u(t) + \frac{1}{L} v(t)$</td>
<td>$h(t)$</td>
</tr>
</tbody>
</table>