REVIEW

1. Principle of Superposition
2. Source Equivalents
3. Thevenin and Norton Equivalent Circuits
4. Computation of a Thevenin Equivalent for Circuits with Independent Sources

REVIEW OF PROCEDURES FOR COMPUTING SOURCE EQUIVALENTS

PROCEDURES FOR THEVENIN EQUIVALENT CIRCUITS

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance $R_{AB}$. This resistance will equal $R_{TH}$

4) Return all Independent Sources to their original values and compute the voltage value corresponding to the voltage drop from A to B, $V_{AB}$ (with Nodes A and B open-circuited). This voltage, $V_{AB} = V_{TH}$

5) You may compute this voltage using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Thevenin Equivalent Circuit.
PROCEDURES FOR NORTON EQUIVALENT CIRCUITS

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance $R_{AB}$. This resistance will equal $R_N$.

4) Return all Independent Sources to their original values and then apply a short circuit at the terminals A and B. Compute the current value from A to B, $I_{AB}$ (with Nodes A and B short-circuited). This current is the Norton Equivalent Current, $I_{AB} = I_N$.

5) You may compute this current using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Norton Equivalent Circuit using this $I_N$ and $R_N$. Note polarities for $I_N$.

NORTON EQUIVALENT CIRCUITS

- The dual of the Thevenin Source Equivalent is the Norton Source Equivalent.

Figure 1. Replacement of a Linear Circuit System with either a Thevenin Source Equivalent or Norton Source Equivalent Circuit containing an Equivalent Resistor. Note that $R_{eq}$ is the same for both circuits.
Let's compute a Thevinin and Norton Source Equivalent for a circuit.

**Figure 2. The Example Circuit**

- Here is the procedure:

**PROCEDURES FOR THEVENIN EQUIVALENT CIRCUITS**

1) Identify and Label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.
2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.
3) Compute the resistance \( R_{AB} \). This resistance will equal \( R_{TH} \)
4) Return all Independent Sources to their original values and compute the voltage value corresponding to the voltage drop from A to B, \( V_{AB} \) (with Nodes A and B open-circuited). This voltage, \( V_{AB} = V_{TH} \)
5) You may compute this voltage using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.
6) Draw the new Thevenin Equivalent Circuit.

- Let's follow this. First, we see that the terminals are labeled.
- Let's remove the voltage source and replace it with its “zero” element – a short circuit.
- Then, let's compute \( R_{AB} \).
- This is simple, \( R_{AB} \) will be a resistance consisting of an 8Ω resistor in series with a parallel combination of a 10Ω and 40Ω resistor. This parallel combination is itself, 8Ω, so \( R_{AB} = R_{TH} = 16Ω \)
- Now, let's compute \( V_{TH} \).
- We return the voltage source to its original value, and compute the voltage at the terminals, a and b.
- Here, we can just use the Voltage Divider equation. Why?
- Now, \( V_{AB} = V_{TH} = 48V \). Thus, our equivalent circuit is:
• Now, let's compute the Norton Equivalent

PROCEDURES FOR NORTON EQUIVALENT CIRCUITS

1) Identify and label the two terminals (A and B) of the circuit system that is intended to be replaced by an Equivalent Source.

2) Replace all Independent Sources with zero elements. Leave Dependent Sources unmodified.

3) Compute the resistance $R_{AB}$. This resistance will equal $R_N$.

4) Return all Independent Sources to their original values and then apply a short circuit at the terminals A and B. Compute the current value from A to B, $I_{AB}$ (with Nodes A and B short-circuited). This current is the Norton Equivalent Current, $I_{AB} = I_N$.

5) You may compute this current using any of the methods we have developed. (Direct application of KCL and KVL, Node Voltage, Mesh Current, or using equivalent resistance methods to simplify the circuit. List known values of circuit variables.

6) Draw the new Norton Equivalent Circuit using this $I_N$ and $R_N$. Note polarities for $I_N$.

• Let's follow this. First, the terminals are labeled.

• We remove the voltage source and replace it with its “zero” element – a short circuit.

• Then, we compute $R_{AB}$.

• We just found this to be $R_{AB} = R_N = 16\Omega$.

• Now, let's compute $I_N$.

• We return the voltage source to its original value, and compute the short circuit current value at the terminals, a and b. Let's examine the circuit that results from this:
Figure 4. Circuit construction for computing the Norton Equivalent Current

- Let's solve for the current flowing through the $8\Omega$ resistor. This is a familiar problem. Let's just compute the voltage drop across the $8\Omega$ resistor. This is just equal to $v_c$.
- First, we can compute the voltage at the node joining the $40\Omega$ and $8\Omega$ resistors. We start by replacing them with their parallel equivalent. This parallel equivalent is just $6.66\Omega$.
- Now, then we can use the Voltage Divider Equation again. The voltage, $v_c$, from this is,

\[ v_c = 60V \frac{6.66\Omega}{16.66\Omega} = 24V \]

- Then, the current through the $8\Omega$ resistor is just $24V/8\Omega = 3A$.
- Thus, $I_N = 3A$. Thus, our equivalent circuit is:

Figure 5. The Norton Source Equivalent for our Example Circuit

- Let's check this result and our understanding.
- First, these two circuits should show the same open circuit output voltage:
The Thevenin Source Equivalent output voltage is 48V – this is easily seen.

The Norton Source Equivalent is also 48V, just the voltage drop due to the 3A current flow through the 16Ω resistor.

- Also, these circuits should show the same short circuit output current.
  - The Thevenin Source Equivalent short circuit current is 3A
  - The Norton Source Equivalent short circuit current is 3A

- These three circuits, the original circuit and the two Thevenin and Norton Equivalents all show exactly the same terminal characteristics.

**SOURCE EQUIVALENT CIRCUITS WITH DEPENDENT SOURCES**

- In the development of amplifier circuits, we will frequently encounter the need for developing Source Equivalent circuits involving dependent sources.

- All of our techniques will apply.

- However, we must introduce one new method for computing resistance. This will be useful to you later, for example in EE115B.

- Here is our circuit (a textbook Drill Exercise):

  ![Figure 6. An Example Circuit for computation of Source Equivalents](image)

- First, let's use the general procedures to compute the Thevenin Source Equivalent.

- We will begin by computing $V_{TH}$
• Lets use the Node Voltage method with a Reference Node where the two Independent Sources and the $8\Omega$ resistor meet.

• Now, note that there is only one Node Equation required. Why?

• At terminal $a$, an Essential Node, we have the KCL equation:

$$\frac{(24V - v_a)}{2} + (-3i_x) - (4A) - \frac{v_a}{8} = 0$$

• Also, the Dependent Source constraint equation is:

$$i_x = \frac{v_a}{8}$$

• This is now easy to solve. Substituting for $i_x$:

$$\frac{(24V - v_a)}{2} + (-3 \cdot \frac{v_a}{8}) - (4A) - \frac{v_a}{8} = 0$$

• and

$$v_a = v_{TH} = 8V$$

• Now, we must compute $R_{AB}$

• There are two approaches:
  
  o 1) 
  
  ▪ One may construct a method where we short circuit the output terminals and compute current in the short circuit, $i_{SC}$
  
  ▪ Then, the resistance, $R_{AB}$, is just $v_{TH}/i_{SC}$
  
  ▪ Lets return to our definitions of the Thevenin Equivalent to see this.

  o 2) 
  
  ▪ An alternative (and our recommend) approach for problems we will encounter, is to compute $R_{AB}$ in a fundamental method:

  ▪ Replace all Independent Sources with their “zero” elements. Note, Dependent Sources remain.

  ▪ Apply a test voltage, $V_T$ to the output terminals

  ▪ This will yield a test current, $I_T$

  ▪ Compute $R_{AB} = V_T/I_T$

  ▪ This is a powerful method that will never let you down.
Let us proceed with the second case.

Let us replace the all Independent Sources with their “zero” elements. Also, let's apply the voltage, $V_T$.

![Circuit Diagram](image)

Figure 7. Our Example Circuit placed in a form required to compute $R_{AB}$.

Let us again use a Node Voltage method to compute $I_T$.

Again, we will place our Reference at the lower node on this diagram where the $8\Omega$, $2\Omega$, and the two sources meet.

Then, the Node Voltage Equation is:

$$\frac{-V_T}{2\Omega} + (-3i_x) - \frac{V_T}{8\Omega} + I_T = 0$$

but,

$$i_x = \frac{V_T}{8}$$

So, our equation becomes:

$$\frac{-V_T}{2\Omega} + (-3\frac{V_T}{8\Omega}) - \frac{V_T}{8\Omega} + I_T = 0$$

Rearranging,

$$\frac{-V_T}{1\Omega} + I_T = 0$$

But, by definition:

$$R_{AB} = \frac{V_T}{I_T} = 1\Omega$$

Thus, our Thevenin Source Equivalent circuit is
• Point for discussion: note that the Thevenin resistance is a very low value. Why? This has application in low input impedance amplifiers.

APPLICATIONS OF EQUIVALENT CIRCUITS

• Frequently, in design or design analysis, we must meet a design specification for a value for “input” or “output” resistance (or impedance) for our circuit system.

• This results from the need to enable specifications for interfaces between components. Interface specifications include both physical node connections as well as electrical specifications.

• Specification of electronic characteristics is best accomplished via source equivalents.

• An example may be the following:
  
  ○ We are asked to supply a voltage of 10V-12V to a resistive load of that will be no less than 1kΩ. (This resistive load may appear at the “input” terminals of a second, large circuit system.) We may also be limited to implementing our circuit with supply potentials no greater than 12V.

  ○ Thus, we would specify that our circuit Thevenin equivalent that would drive this load will be 12V, and the Thevenin resistance, would be no greater than $R_{TH} = 200\, \Omega$.

• For many examples, we wish to select a circuit resistive load value such that our circuit system transfers the largest power to the load.
To address this problem, we will use a Thevenin Equivalent.

![Thevenin Equivalent Diagram](image)

**Figure 8** A Thevenin equivalent source and load resistor.

- Consider the circuit in Figure 2. Here, a load resistor, $R_{\text{Load}}$ is applied to a circuit structure arranged as a Thevenin equivalent, voltage source and series resistor pair.

- We may compute the power dissipated in $R_{\text{Load}}$. This will be

$$P_{\text{Absorbed}} = \frac{v_o^2}{R_{\text{Load}}}.$$

- However, we may compute $v_o$ directly with the voltage divider equation. So,

$$P_{\text{Absorbed}} = \frac{v_o^2}{R_{\text{Load}}} = \frac{V_{\text{TH}}^2 R_{\text{Load}}}{(R_{\text{Load}} + R_{\text{TH}})^2}.$$

- Now $P_{\text{Absorbed}}$ depends on $R_{\text{Load}}$. We would like to find the value of $R_{\text{Load}}$ such that $P_{\text{Absorbed}}$ is maximum.

- So, to compute the maximum value, let’s begin by computing the derivative of power, $P_{\text{Absorbed}}$ with respect to $R_{\text{Load}}$. This is

$$\frac{dP}{dR_{\text{Load}}} = V_{\text{TH}}^2 \left[ \frac{1}{(R_{\text{Load}} + R_{\text{TH}})^2} - 2R_{\text{Load}}(R_{\text{Load}} + R_{\text{TH}})/(R_{\text{Load}} + R_{\text{TH}})^4 \right].$$

- Now, for non-zero $V_{\text{TH}}$, this derivative is zero at a value of $R_{\text{Load}}$ where,

$$\frac{1}{(R_{\text{Load}} + R_{\text{TH}})^2} - 2R_{\text{Load}}(R_{\text{Load}} + R_{\text{TH}})/(R_{\text{Load}} + R_{\text{TH}})^4 = 0$$

- or

$$[(R_{\text{Load}} + R_{\text{TH}})^2 - 2R_{\text{Load}}(R_{\text{Load}} + R_{\text{TH}})]/(R_{\text{Load}} + R_{\text{TH}})^4 = 0$$

- or

$$(R_{\text{Load}} + R_{\text{TH}})^2 - 2R_{\text{Load}}(R_{\text{Load}} + R_{\text{TH}}) = 0 \text{ and } 2R_{\text{Load}} = (R_{\text{Load}} + R_{\text{TH}})$$

- So, maximum power transfer occurs at $R_{\text{Load}} = R_{\text{TH}}$

$$R_{\text{Load Max}} = R_{\text{TH}}$$
• So, this frequently used result states that the maximum power transfer to a load from a fixed Thevenin circuit equivalent, occurs when the load resistor value equals the Thevenin resistance value.

• Also, the specific value of the maximum power can be computed from the above results, with

\[ P_{\text{Absorbed Max}} = \frac{V_O^2}{R_{\text{Load Max}}} = \frac{V_{\text{TH}}^2 R_{\text{TH}}}{(R_{\text{TH}} + R_{\text{TH}})^2} \]

\[ \text{or} \]

\[ P_{\text{Absorbed Max}} = \frac{V_{\text{TH}}^2}{4R_{\text{TH}}} = \frac{V_{\text{TH}}^2}{4R_{\text{Load Max}}} \]