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HOMEWORK ASSIGNMENT DUE WEDNESDAY, JANUARY 28, BEFORE CLASS
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REVIEW

1) Fundamental Resistor Circuits
   a. Series Resistor Circuits
   b. Parallel Resistor Circuits

2) The Voltage Divider

3) The Current Divider

4) The Digital to Analog Converter

5) The Wheatstone Bridge Circuit

6) Resistor Circuit Transformations
CIRCUIT ANALYSIS: THE NODE-VOLTAGE AND MESH-CURRENT METHODS

- Given our background with Kirchhoff’s Laws, Ohm’s Law, and the series and parallel equivalent circuit methods, we are now prepared to introduce a set of new analysis methods.
- First, we need a set of rules that provide the guidance for selecting the circuit paths that produce the most rapidly obtained solution.
- An important circuit concept of a reference potential leads to a new analysis method, the node-voltage method. This is used throughout circuit analysis and is important in the intuition for circuit design.
- The node-voltage analysis method, as we will see, is particularly applicable to systems that contain many parallel elements.
- We will also learn about the mesh-current method. This is a powerful technique that is applicable to circuits that contain many series elements.
- As we make progress, we will become increasingly capable of solving circuit problems and developing the intuition and visualization for circuit design.

IMPORTANT DEFINITIONS OF CIRCUIT FEATURES

- A set of new circuit rules will rely on definitions of:
  - nodes (connecting 2, 3, or more elements)
  - and paths (open and closed).
- This set of definitions is shown below.

<table>
<thead>
<tr>
<th>Node</th>
<th>The intersection between two or more circuit elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Essential Node</td>
<td>The intersection between three or more elements</td>
</tr>
<tr>
<td>Path</td>
<td>A continuous line segment passing through adjacent circuit elements that passes through circuit elements only once.</td>
</tr>
<tr>
<td>Branch</td>
<td>A path that connects two nodes</td>
</tr>
<tr>
<td>Essential Branch</td>
<td>A path that connects two essential nodes without crossing other essential nodes</td>
</tr>
<tr>
<td>Loop</td>
<td>A closed path – a path that starts and ends at the same node without passing through any other node more than once</td>
</tr>
<tr>
<td>Mesh</td>
<td>A loop that does not enclose any other loops. This important term derives from the appearance of circuits that are drawn with these loops superimposed.</td>
</tr>
</tbody>
</table>
The graphical computation rules we will use will exploit a set of graphical constructions that require that all components of a circuit be drawn such that they lie on a plane and where no elements or conductors intersect.

These are so-called planar circuits. Figure 1a shows a circuit drawn with intersecting conductors. It appears to be non-planar. However, redrawing a conductor, arriving at 1b, shows the circuit is planar.

In contrast, Figure 1c shows a circuit that appears non-planar. Redrawing a conductor still yields a structure where an intersection is “trapped”. This circuit is non-planar and the mesh-current tools we will discuss are not applicable.

Now, to illustrate the definitions, we will consider the following circuit, in Figure 2.

![Figure 2. Circuit example used for illustration of circuit feature definitions.](image-url)
We will take each definition in turn and list results in the table below.

<table>
<thead>
<tr>
<th>Nodes</th>
<th>a – m. Now, n and o are not unique nodes (why?)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Essential Nodes</td>
<td>e, h, j, m</td>
</tr>
<tr>
<td>Branches</td>
<td>All branches include $R_1$ through $R_{11}$, and sources $v_{S1}$ and $v_{S2}$, and $i_s$</td>
</tr>
<tr>
<td>Essential Branches</td>
<td>All essential branches</td>
</tr>
<tr>
<td></td>
<td>$R_1 - R_2 - R_3 - R_4 - v_{S2}$</td>
</tr>
<tr>
<td></td>
<td>$R_5 - v_{S1} - R_6$</td>
</tr>
<tr>
<td></td>
<td>$i_s$</td>
</tr>
<tr>
<td></td>
<td>$R_{10}$</td>
</tr>
<tr>
<td></td>
<td>$R_7 - R_8 - R_9$</td>
</tr>
<tr>
<td></td>
<td>$R_{11}$</td>
</tr>
<tr>
<td>Loops</td>
<td>Some loops include</td>
</tr>
<tr>
<td></td>
<td>$R_1 - R_2 - R_3 - R_4 - v_{S2} - R_6 - v_{S1} - R_5$</td>
</tr>
<tr>
<td></td>
<td>$R_1 - R_2 - R_3 - R_4 - v_{S2} - R_{10} - R_{11} - i_s$</td>
</tr>
<tr>
<td></td>
<td>$R_5 - v_{S1} - R_6 - R_{10} - R_{11} - i_s$</td>
</tr>
<tr>
<td>Meshes</td>
<td>All meshes include</td>
</tr>
<tr>
<td></td>
<td>$R_1 - R_2 - R_3 - R_4 - v_{S2} - R_6 - v_{S1} - R_5$</td>
</tr>
<tr>
<td></td>
<td>$R_7 - R_8 - R_9 - R_{11}$</td>
</tr>
<tr>
<td></td>
<td>$R_5 - v_{S1} - R_6 - R_{10} - R_9 - R_8 - R_7 - i_s$</td>
</tr>
</tbody>
</table>

Now, note that all branches, loops, and meshes are paths.

Let us list examples of paths that are not essential branches

<table>
<thead>
<tr>
<th>Paths that are not essential branches</th>
<th>e, j, k, l, m, h (Not an essential branch since this passes through more than one essential node)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_2 - R_3 - R_4 - v_{S2}$ (Not an essential branch since this does not terminate on essential nodes)</td>
</tr>
</tbody>
</table>
THE NODE-VOLTAGE CIRCUIT ANALYSIS METHOD INTRODUCTION

- Now, armed with these definitions, we can proceed to introduce a powerful analysis method, the node-voltage method.
- First, let's review the requirements on the number of equations we require to solve a circuit problem.
- First, if the number of branches is $N_{\text{Branches}}$, where the current is unknown, then we require $N_B$ simultaneous equations to solve for these currents.
- If the number of nodes is $N_{\text{Nodes}}$, then we can derive $(N_{\text{Nodes}} - 1)$ independent equations from KCL.
- So, in general, $(N_{\text{Nodes}} - 1)$ is less than $N_{\text{Branches}}$.
- We therefore need, $N_{\text{Branches}} - (N_{\text{Nodes}} - 1)$ additional KVL equations to solve our circuit problem.
- The Node-Voltage method introduces a means to solve a circuit problem considering equations depending on the parameters of only Essential Nodes.
- (It combines information from KVL and Ohm's Law equations with KCL)
- Then, the number of equations we must write down is now required to be $N_E - 1$
- This is a potentially major reduction in complexity since the number of Essential Nodes is generally much less than the total number of nodes.
- The Node Voltage Method requires definition of a Reference Potential.

PROBLEM SOLVING APPROACH WITH NODE VOLTAGE METHOD

1) List the information that is requested by the problem.
2) Examine the circuit to determine the approach for a solution
   a. Determine if a circuit simplification may be accomplished using an equivalent circuit, for example, a parallel, series, delta, or Y, circuit structure.
3) List known values of circuit variables
4) Identify and label the $N_E$ Essential Nodes
5) Choose one Essential Node and label it with a Reference Potential Symbol. The choice of this node will determine the level of simplicity of the calculation. However, any choice of an Essential Node will yield the same problem results. You should select in the circuit, that Essential Node that is connected to the most branches.
6) Identify and label the non-reference node voltages.

7) Each non-reference node voltage is labeled as positive.

8) Use KCL to write down an equation for each non-reference node, writing the equations in terms of the resistances, and node voltages.

9) Write down \( N_E - 1 \) equations.

10) Solve the set of equations.

- Here is an example of a part of a circuit problem solution, focusing on a pair of nodes. We have reached Step 8 in the procedure above in this example of a calculation.

- Note, we have not labeled currents in the usual way at this point – this is the Node Voltage Method!

![Figure 4](image_url)

- Now, with the tools we have, we may immediately write down the value of the currents \( i_{12} \) and \( i_{23} \)

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*Figure 3. Reference Potential Symbol*

*Figure 4. A segment of a circuit, labeled with Node Voltages and Reference Potential, as well as labeled branch currents.*
• First, let's consider the current between nodes 1 and 2. This is just depends on the voltage difference between 1 and 2.

• But, we can write this down immediately. This is just $v_1 - v_2$.

• So, we can write down the current immediately using Ohm’s Law:

$$i_{12} = \frac{v_1 - v_2}{R_3}$$

• Also, the current through $R_4$ is

$$i_{23} = \frac{v_2 - v_3}{R_4}$$

• Finally, let's examine node 2. The current through $R_2$, leaving node 2 and arriving at the Reference Node, $i_{2R}$, is just computed by Ohm’s Law:

$$i_{2R} = \frac{v_2}{R_2}$$

• Now, we can apply Kirchhoff’s Current Law at node 2. We will sum currents arriving at a node to be positive contributions to the KCL sum, as usual.

• What are the current directions? They are just determined by the Passive Sign Convention and Ohm’s Law.

• So, at node 2 we have:

$$(i_{12}) + (-i_{2R}) + (-i_{23}) = 0$$

• Or,

$$\frac{v_1 - v_2}{R_3} - \frac{v_2}{R_2} - \frac{v_2 - v_3}{R_4} = 0$$

• We can illustrate this with a circuit solution (let's take the Drill Exercise, 4.5. Note that the circuit labeling is different than in the text. $i_j$ is replaced with $i_{12}$)
EXAMPLE NODE-VOLTAGE CIRCUIT SOLUTION

Figure 5. Circuit for Example Node Voltage Problem. Note there are some differences in labeling here relative to the textbook.

- Using previous methods, we would expect to encounter 8 simultaneous equations for this problem. Let us proceed with the Node Voltage method now and compare.

- We will follow our procedure:

  1) List the information that is requested by the problem.
     a. \( v_1, v_2, i_{12} \)
     b. Power delivered by the sources – (note, we will watch our polarities !! )

  11) Examine the circuit to determine the approach for a solution
     a. We will use the Node Voltage method and make no other modifications for now.

  12) List known values of circuit variables
     a. We have the two source currents.

  13) Identify and label the \( N_E \) Essential Nodes
     a. We have (3) three Essential Nodes

  14) Choose one Essential Node and label it with a Reference Potential Symbol. The choice of this node will determine the level of simplicity of the calculation. However, any choice of an Essential Node will yield the same problem results. You should select in the circuit, that Essential Node that is connected to the most branches.
      a. The node joining the two sources has five branches. This is the best choice as a reference potential.
15) Identify and label the non-reference node voltages.

16) Each non-reference node voltage is labeled as positive.
   a. And here is the new, labeled circuit.

17) Use KCL to write down an equation for each non-reference node, writing the equations in terms of the resistances, and node voltages.

18) Write down $N_{E} - 1$ equations.
   a. This is straightforward.

   Node 1) \[
   (15) + \left( -\frac{v_1}{60} \right) + \left( -\frac{v_1}{15} \right) + \left( -\frac{v_1 - v_2}{5} \right) = 0
   \]

   Node 2) \[
   \left( \frac{v_1 - v_2}{5} \right) + \left( -\frac{v_2}{2} \right) + (-5) = 0
   \]

19) Solve the set of just two equations!
   a. $v_1 = 60V$, $v_2 = 10V$ and $i_{12} = (v_1 - v_2)/5 = (60V - 10V)/5\Omega = 10A$
   b. Power for 15A source
      i. Lets discuss this.
ii. We see the polarity of the potential reference follows the Active Sign Convention. So, \( p = -iv_1 = -(15\text{A})*(60\text{V}) = -900\text{W} \). Thus, power absorption is negative. Power being delivered is +900W.

iii. We can also conclude this by noting that the derived voltage value, \( v_1 \) is positive, indicating the current is leaving at the positive (high) potential reference. Thus, this element must be delivering power.

c. Power for 5A source.

i. We see the polarity of the potential reference follows the Passive Sign Convention. So, \( p = iv_2 = (5\text{A})*(10\text{V}) = 50\text{W} \). Thus, power absorption is a positive 50W. Power being delivered is –50W.

ii. We can also conclude this by noting that the derived voltage value, \( v_2 \) is positive, indicating the current is arriving at the positive (high) potential reference. Thus, this element must be absorbing power or showing a negative power delivery.