1. In the above RC circuit, the switch is closed at time $t = 0$, with the capacitor initially discharged. After that, the sinusoidal voltage $x(t) = \cos(2\omega_0 t)$ is applied, where $\omega_0 = \frac{1}{RC}$. Find the output voltage $y(t)$.

2. A linear, time invariant system has impulse response function given by

$$h(t) = a \delta(t) + b e^{-t} u(t) + c t e^{-t} u(t)$$

where $a, b, c$ are constants. We are given the following information:

- When the input is $x(t) \equiv 1$ for $t \in (\infty, \infty)$, the output is the same as the input.
- When the input is $x(t) = \cos(t)$ for $t \in (\infty, \infty)$, the output is zero.

a) Find $a, b, c$.
b) Now let the input be $x(t) = \cos(t) u(t)$. Find the output.

3. For the following functions, find out whether they are periodic, and if so, give the period.

a) $\cos(2t) + \sin(5t - \pi)$; b) $(\sin(t))^2$; c) $\sin(t^2)$. 
1. Consider the function of period $T = 2$ defined by

$$f(t) = \begin{cases} t & \text{for} \ 0 \leq t \leq 1, \\ 0 & \text{for} \ 1 \leq t \leq 2 \end{cases}$$

a) Find the Fourier series representation $\sum_{n=-\infty}^{\infty} F_n e^{i\omega_0 t}$.
b) Find the sine-cosine Fourier series representation.
c) Compute $\sum_{n=-\infty}^{\infty} |F_n|^2$.

2. Given a periodic function $f(t)$ with Fourier series expansion $f(t) = \sum_{n=-\infty}^{\infty} F_n e^{i\omega_0 t}$.

   a) Differentiate the above to derive a formula for the Fourier series of $\frac{df}{dt}(t)$.
   b) Now consider $f(t)$ of period $T = 3$, defined by

$$f(t) = \begin{cases} -1 - t & \text{for} \ t \in [-1, 0] \\ 2 - t & \text{for} \ t \in [0, 2] \end{cases}$$

Sketch $f(t)$, $\frac{df}{dt}(t)$, and find their Fourier series expansions.

3. Consider the rectifier system defined by the equation

$$y(t) = \max\{x(t), 0\} = \begin{cases} x(t) & \text{if} \ x(t) > 0, \\ 0 & \text{otherwise}. \end{cases}$$

We excite it with an input $x(t) = V_0 \sin(\omega_0 t)$.

   a) Sketch the output $y(t)$ and find its fundamental frequency.
   b) Find the Fourier series expansion of $y(t)$.
   c) Find the total power, DC power and AC power of the signal $y(t)$.
1. Find the Fourier series expansion of \( f(t) = (\cos(\omega_1 t))^4 \), both in complex exponential and in sine-cosine forms. *Hint:* no need to compute any integrals!
   Also find the minimum mean-square error when approximating \( f(t) \) by a sum of sinusoids of frequencies up to \( 2\omega_1 \).

2. This problem follows up on Hw6, Problem 3. The rectifier block \( \mathcal{R} \) in the figure follows the equation \( y(t) = \max\{x(t), 0\} \) as before, and \( x(t) = V_0 \sin(\omega_0 t) \). We now add a “lowpass” RC circuit in cascade to further attenuate the AC components, similarly to what was done in class for a different rectifier.
   a) Find the Fourier series expansion of the output voltage \( z(t) \), and its DC power.
   b) Design the time-constant \( \tau = RC \) of the circuit so that the AC power of \( z(t) \) is at least a factor of 10 smaller than that of \( y(t) \).

3. Find the Fourier transforms of the following functions:
   a) \( f(t) = te^{-|t|} \).
   b) \( f(t) = \begin{cases} 1 & \text{for } |t| < a. \\ 0 & \text{otherwise} \end{cases} \)

4. Find the inverse Fourier transforms of the following:
   a) \( F(i\omega) = (\omega + 1)[u(\omega + 1) - u(\omega)] + (1 - \omega)[u(\omega) - u(\omega - 1)]. \)
   b) \( F(i\omega) = \sin(T\omega) \).
1. We are given an LTI system with frequency response function $H(i\omega)$, and we apply to it an input with Fourier transform $X(i\omega)$. Both are plotted below.

a) Find the impulse response $h(t)$.

b) Find the input $x(t)$.

c) Find the output $y(t)$.

2. Based on the Fourier transform $\mathcal{F}[u(t+1) - u(t-1)] = \frac{2\sin(\omega)}{\omega}$, use Fourier properties to derive the transforms of the following signals. It may be useful to first sketch the signals.

a) $f(t) = u(t + b) - u(t + a) + u(t - a) - u(t - b)$.

b) $f(t) = u(t - a) - u(t - b)$.

c) $f(t) = [u(t + 1) - u(t - 1)] \cos(\pi t)$.

d) $f(t) = (t + 1)u(t + 1) - (t - 1)u(t - 1)$.

3. Use Parseval’s relation to compute

$$\int_{-\infty}^{\infty} \frac{\sin^2(t)}{t^2} dt.$$
4. In the system below

- $H_{low}$ is an ideal low-pass filter, with cutoff frequency $\omega_0$.
- $u(t) = x(t) \cos(\omega_0 t)$ and $y(t) = v(t) \cos(\omega_0 t)$.
- $x(t)$ is band-limited to $[-B, B]$, as depicted in the figure in the next page. $X(0) = A$.
- $\omega_0 > 2B$.

Sketch the Fourier transforms $U(i\omega)$, $V(i\omega)$, $Y(i\omega)$ and $Z(i\omega)$, and relate $z(t)$ to $x(t)$. Justify your answer.