1. Review of integration.
   (a) Use integration by parts to evaluate the integrals \( \int_0^1 \arctan(t) dt \) and \( \int_0^\infty e^{-t^2} t^3 dt \).
   (b) For a continuous function \( f \), let
   \[
   g(t) = \int_0^t f(t - \tau) d\tau.
   \]
   Find \( \frac{dg}{dt} \).
   (c) The figure below contains a sketch of a function \( f(t) \); the function is zero outside the interval \([0, 3\pi]\). Find the function \( g(t) = \int_{-\infty}^t f(\tau) d\tau \) and sketch it under \( f(t) \).

2. Review of complex numbers
   (a) Find the following complex numbers (real and imaginary parts):
   \[
   \begin{align*}
   (1) & \quad i e^{-\frac{9}{4} \pi i}, \quad (2) \quad (-i)^{-3}
   \end{align*}
   \]
   (b) Change these complex numbers into exponential form:
   \[
   \begin{align*}
   (1) & \quad \alpha = 1 + \sqrt{3}i, \quad (2) \quad \beta = \sqrt{3} + i.
   \end{align*}
   \]
   (c) For the numbers in part (b), compute \( \alpha/\bar{\beta} \), where \( \bar{\beta} \) is the complex conjugate of \( \beta \).
   (d) Find the complex roots of the polynomial equation \( z^4 + z^2 + 1 = 0 \).
3. Solve the differential equation

\[ \frac{dy(t)}{dt} + y(t) = t, \quad y(0) = a. \]

4. For each of the following systems with input \( x(t) \) and output \( y(t) \), find out whether they are (i) linear, (ii) time invariant, (iii) causal. Justify your answer.

(a) \( y(t) = \int_{t}^{t+1} x(\sigma)d\sigma \).

(b) \( y(t) = x(t) - [x(t)]^2 \).

(c) \( y(t) = x(t - t^2) \).

(d) \( y(t) = x(t) - t^2 \).
1. Sketch $f(t)$ and $\frac{df}{dt}(t)$. State what $\frac{df}{dt}(t)$ is in the simplest form (e.g., $u(t-2)\delta(t-7)$ should be simplified to $\delta(t-7)$).
   (a) $f(t) = u(t) + u(t+1) - 2u(t-2)$.
   (b) $f(t) = \begin{cases} 
   1 - t^2 & \text{for } t \in (-1, 0) \\
   -1 + t^2 & \text{for } t \in (0, 1) \\
   0 & \text{otherwise}
   \end{cases}$. Here you should first write an expression for $f(t)$.
   (c) $f(t) = (t + 2)[u(t + 1) - u(t)] + (2 - 2t)[u(t) - u(t - 2)]$.

2. Sketch $\frac{df}{dt}(t)$, and find an expression for $f(t)$ and $\frac{df}{dt}(t)$.

(a)

(b)
3. Evaluate the following integrals.

(a) \[ \int_{-\infty}^{\infty} t^3 \cos(\pi t) \, \delta(t - 1) \, dt \]

(b) \[ \int_{0}^{\infty} \sqrt{t + 1} e^t \, \delta(t + 2) \, dt \]

(c) \[ \int_{-\infty}^{t^+} \cos(\sigma) \delta(t - \sigma) \, d\sigma. \]

4. Consider the system defined by the input-output relationship

\[ y(t) = \int_{-\infty}^{t+1} (t - \sigma + 2)^2 x(\sigma) d\sigma. \]

(a) Find the system impulse response function \( h(t, \tau). \)

(b) Is the system time invariant? Causal?

5. Consider a system defined by the differential equation

\[ \frac{dy(t)}{dt} + y(t) = tx(t), \]

for \( t \geq 0, \) with initial condition \( y(0) = 0. \) For this system we consider only positive times.

(a) Find the system impulse response function \( h(t, \tau). \)

(b) Is the system time invariant? Causal?
1. We consider a linear, time-invariant system with impulse response $h(t)$ depicted in the figure. The function $h(t)$ is non-negative, it is zero outside the interval $[0, 1]$, and $\int_0^1 h(t)dt = 1$.

Sketch the response of the system to the input $x(t) = u(t) - u(t - 1) - u(t - 2) + u(t - 4)$. Your sketch cannot be exact since you don’t know $h(t)$ exactly, but it must be consistent with the information given above.

2. Given the functions: $f(t) = \cos(t)$, $g(t) = u(t) - u(t - \pi)$ and $h(t) = u(t) - u(t - 2\pi)$; find the convolutions:
   
   (a) $f \ast g$  
   (b) $f \ast h$  
   (c) $g \ast h$

3. We consider a linear, time-invariant system $S_1$. We know that applying to the system the test input $x(t) = u(t)e^{-t}$, the output is $y(t) = u(t)(1 - e^{-t})$.

   (a) Find the impulse response $h_1(t)$ of the system $S_1$. * Hint: consider the response to the derivative of the given input. 

   (b) Now we connect the system in cascade with $S_2$, that is also LTI with impulse response $h_2(t) = u(t) - 2u(t - 1) + u(t - 2)$. Find and sketch the impulse response of the cascade.

4. Use the definition of the Laplace transform

$$F(s) = \int_0^\infty e^{-st} f(t)dt$$

to find the transform of the following functions. In each case specify the domain of convergence.

(a) $u(t) \sin(\omega t + \phi)$, where $\omega, \phi$ are constants.

(b) $(t + 1)u(t - 1)$.

(c) $u(t) - \delta(t) - u(t - 2)$. 
1. Find the Laplace transforms of the following functions using the properties of Laplace transform. Specify the properties being used, and the DOC.

(a) \( \int_0^t \tau \cos(\tau) d\tau \).
(b) \( e^t u(t-2) \cdot (t-2)^2 \).

2. Given the function \( f(t) = \sin(t) \left( u(t) - u(t-3\pi) \right) \).
   a. Find the derivatives \( \frac{df}{dt}, \frac{d^2f}{dt^2} \), and deduce a simple formula relating \( f(t) \) with \( \frac{d^2f}{dt^2} \).
   b. Taking Laplace transforms in the previous formula, find the transform \( F(s) \). You should not need to compute any integrals. Specify the domain of convergence.

3. Find \( f(t) \) given \( F(s) \).
   a) \( F(s) = \frac{6(s-1)}{s^2 + 2s - 8} \);
   b) \( F(s) = \frac{s-4}{s^3 + 4s^2 + 4s} \);
   c) \( F(s) = \frac{2s^2 + 6}{(s^2 - 2s + 5)(s + 1)} \).

4. Consider the differential equation for \( t \geq 0 \):
   \[ \frac{d^2f}{dt^2} + 2 \frac{df}{dt} + f(t) = 1, \quad f(0^-) = \alpha, \quad \frac{df}{dt}(0^-) = 0. \]
   Here \( \alpha \in \mathbb{R} \) is a parameter.
   a) Find the initial value \( \lim_{t \to 0^+} f(t) \). Hint: you don’t need to solve the differential equation.
   b) Repeat the above for the final value \( \lim_{t \to +\infty} f(t) \).
   c) Now let \( \alpha = 1 \); solve the differential equation.

5. The response of a linear time invariant causal system to the input \( x(t) = u(t) \cos(t) \) is the output \( y(t) = u(t)e^{-t}(1 - t) \). Find the system impulse response.