Definition of static case:
when all the charges are permanently fixed in space
or if they move, they do so at a steady rate so that
\( \mathbf{p} \) and \( \mathbf{J} \) are constant in time.
\[ \mathbf{J} = \int_\sigma \mathbf{E} \cdot d\mathbf{a} \]
\[ q = \int \mathbf{p} \cdot d\mathbf{v} \]

4.3 Coulomb's Law
\[ \mathbf{E} = \frac{q}{4\pi \varepsilon_0 R^2} \quad (\text{V/m}) \]
\[ \mathbf{D} = \varepsilon \mathbf{E}, \quad \varepsilon = \varepsilon_r \varepsilon_0, \quad \varepsilon_0 = 8.85 \times 10^{-12} \text{C}^2/(\text{N}\cdot\text{m}^2) \]
\[ \text{dielectric constant (relative permittivity)} \]

\[ \mathbf{E} = \frac{1}{4\pi \varepsilon_0} \int_\mathcal{V} \mathbf{E} \cdot \mathbf{R} \frac{P \cdot d\mathbf{v}}{R^2} \quad (\text{volume charge distribution}) \]

\[ \mathbf{E} = \frac{1}{4\pi \varepsilon_0} \int_\sigma \mathbf{E} \cdot \mathbf{R} \frac{P \cdot d\mathbf{s}}{R^2} \quad (\text{surface charge distribution}) \]

\[ \mathbf{E} = \frac{1}{4\pi \varepsilon_0} \int_\mathcal{L} \mathbf{E} \cdot \mathbf{R} \frac{P \cdot d\ell}{R^2} \quad (\text{line charge distribution}) \]
4-4. Gauss's Law
\[ \oint_S \mathbf{D} \cdot d\mathbf{s} = Q \]

Divergence theorem gives,
\[ \oint_S \mathbf{D} \cdot d\mathbf{s} = \int_V \nabla \cdot \mathbf{D} \, dv \]
\[ \Rightarrow \nabla \cdot \mathbf{D} = \frac{Q}{V} \]  

V arbitrary \[ \Rightarrow \nabla \cdot \mathbf{D} = \rho \text{.} \text{ (Gauss's Law in differential form)} \]

Gauss's Law can be used to find electric field from charge distribution for special cases.

Coulomb's Law can be applied for any cases; however, it may require complicate integration process.
(They agree with each other.)
4-5. Electric Scalar Potential.

\[ V_{21} = V_2 - V_1 = - \int_{P_1}^{P_2} \vec{E} \cdot d\vec{l} \]

Energy accumulation when moving a unit charge from \( P_1 \) to \( P_2 \).

\[ \int_S (\nabla \times \vec{E}) \cdot d\vec{s} = \oint_C \vec{E} \cdot d\vec{l} = 0 \quad \text{(Electrostatics)} \]

with Stoke's theorem.

Potential due to point charges:

\[ \vec{E} = \frac{q}{4\pi \varepsilon R^2} \quad \text{(V/m)} \]

\[ V = -\int_\infty^R \frac{d\rho}{R} \]

For continuous distributions of charges:

\[
\begin{align*}
V(R) &= \frac{1}{4\pi \varepsilon} \int V \frac{P}{R} \, dV' \quad \text{(volume distribution)} \\
V(R) &= \frac{1}{4\pi \varepsilon} \int_S \frac{P}{R} \, dS' \quad \text{(surface distribution)} \\
V(R) &= \frac{1}{4\pi \varepsilon} \int_L \frac{P}{R} \, dl' \quad \text{(line distribution)}
\end{align*}
\]
4-5.2. Electric Potential due to Point Charges
\[ E = \hat{R} \frac{q}{4\pi \varepsilon R^2} \quad (\text{V/m}) \quad \text{(Coulomb's Law)} \]
\[ V = -\int_{\infty}^{R} \vec{E} \cdot d\vec{l} = -\int_{\infty}^{R} \left( \hat{R} \frac{q}{4\pi \varepsilon R^2} \right) \cdot d\hat{R} = \frac{q}{4\pi \varepsilon R} \]

For continuous distributions of charges,
\[
\begin{align*}
V(R) & = \frac{1}{4\pi \varepsilon} \int \frac{P_v}{R'} \, dv' \quad \text{(Volume distribution)} \\
V(R) & = \frac{1}{4\pi \varepsilon} \int \frac{P_s}{R'} \, dS' \quad \text{(Surface distribution)} \\
V(R) & = \frac{1}{4\pi \varepsilon} \int \frac{P_l}{R} \, dl' \quad \text{(Line distribution)}
\end{align*}
\]

4-5.4. Electric Field as a Function of Electric Potential
\[ V = \int_{0}^{p} dv = -\int_{0}^{p} \vec{E} \cdot d\vec{l} \Rightarrow dv = -\vec{E} \cdot d\vec{l} \]
According to definition,
\[ dv = \nabla V \cdot dl \quad \text{so it gives} \]
\[ \vec{E} = -\nabla V \]
4-5.5. Poisson's Equation

\[ \nabla \cdot \vec{D} = \frac{P}{\varepsilon} \quad (\text{Gauss's Law}) \]
\[ \vec{D} = \varepsilon \vec{E} \]
\[ \Rightarrow \nabla \cdot \vec{E} = \frac{P}{\varepsilon} \]
\[ \vec{E} = -\nabla \psi \]

Since \[ \nabla \nabla \psi = \nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2} \]

\[ \nabla^2 \psi = -\frac{P}{\varepsilon} \quad (\text{Poisson equation}) \]

For source free region, \( P(x, y, z) = 0 \).

We have \[ \nabla^2 \psi = 0 \quad (\text{Laplace's equation}) \]
1. Transmission line lumped element model (lossless case only)

2. Reflection coefficient.
   - Generally complex
   - (Traveling wave, standing wave, pure standing wave, standing wave ratio)

3. Input Impedance
   - $Z_{in}$; how does it relate to $Z_L$?
   - What physical meaning?
   - Impedance match
4. Smith Chart.
   a. Which complex plane it lies on?
   b. What parameter values can you find out using the chart? E, Zin, Zm, P, SWR.
   c. What does it correspond when the observation point move along the transmission line? (move away from the load -> rotate clockwise)

5. Vector calculus.
   Stoke's theorem
   Divergence theorem