2-10 Impedance Matching

If the load impedance $Z_L$ is equal to $Z_0$, there is no reflection at the load ($\Gamma_L = 0$) $\iff$ Matched condition.

In general, $Z_L \neq Z_0$. Then insert a Matching Network.

Objective: $Z_{in} = Z_0$. Note that $\Gamma_s \neq 0$ unless $Z_g = Z_0$.

Case I: $Z_L = R_L$ (Resistive Load)

Note that $Z_0$ is a real value.

Use a $\lambda/4$ impedance transformer.

\[ Z_{in} = \frac{Z_0}{Z_L} = Z_0 \]

Choose $Z_{01} = \sqrt{Z_0Z_L}$
Case II General Load $Z_L = R_L + jX_L$

Approach: Single-stub matching. (Stub is a transmission line branch)
Use two steps (Transformer and Stub)

Type I Shunt (Parallel) Stub (Open or Short Circuit)

(1) Select the distance $d$ to match the real part.
   $Y_L = 1/Z_L \quad \rightarrow \quad Y_d = Y_0 + jB$

(2) Change the stub length $l$ such that the input admittance of the stub cancel $jB$
   $Y_S = -jB$
   $B$ can be found on Smith chart.
1. At A, $z_L = 0.5 - j1$ ($Z_L = 25 - j50 \, \Omega$)
2. $y_L = 1/z_L = 0.4 + j0.8$ is given at B
3. SWR circle intersect with $g_d = 1$ at C and D.
4. B to C rotation takes $d_1 = 0.063 \lambda$. $y_d = 1 + j1.58$
5. For shorted stub, $l_1 = 0.09 \lambda$ from E (short) to F provides $y_s = -j1.58$.
6. Parallel combination of $y_d$ and $y_s$ makes $y_{in} = 1$.
   $(y_d + y_s = 1 + j1.58 - j1.58)$

What if we use an open stub?
Type II Series Stub (Open or Short)

(1) Select the distance $d$ to match the real part.
   \[ Z_L \rightarrow Z_d = Z_0 + jX \]

(2) Change the stub length $l$ such that the input impedance of the stub cancel $jX$
   \[ Z_S = -jX \]
   $X$ can be found on Smith chart.
1. At A, $z_L = 0.5 - j 1$ ($Z_L = 25 - j 50 \, \Omega$)
2. SWR intersects with $r_d = 1$ at C and D.
3. A to C rotation takes $d_1 = (0.25 + 0.063)\lambda$.
   $$z_d = 1 + j 1.58$$
4. For an open stub, $l = 0.09 \lambda$ from E (open) to F provides $z_s = - j 1.58$.
5. Series combination of $z_d$ and $z_s$ provides $z_{in} = 1$.
   $$(z_{in} = z_d + z_s)$$
Type III Transformer Type

No stub but two sections of transformers

\[ Z_\text{in} = R_{\text{in}} \]

(1) Select \( d_1 \) such that \( Z_{\text{in}} \) is real. \( (Z_{\text{in}} = R_{\text{in}}) \)
(2) Use a \( \lambda/4 \) transformer to match to \( Z_0 \).
1. At A, $z_L = 0.5 - j1$.
2. SWR intersects with $x_L = 0$ line at $P_{\text{min}}$ and $P_{\text{max}}$.
3. A to $P_{\text{min}}$ is shorter ($d_1 = (0.50 - 0.365)\lambda$)
4. At $P_{\text{min}}$, $z_{in} = 0.235 + j0$.
5. Place a $\lambda/4$ transformer $z_2 = \sqrt{z_{in}z_0}$