1. A relaxed system is described by the difference equation

\[ y(n) - \frac{1}{2} y(n-1) = x^2(n), \quad n \geq 0 \]

where \( x(n) \) denotes the input sequence and \( y(n) \) denotes the output sequence. Prove or give counter-examples:

(a) (10 PTS) Is the system linear?
(b) (10 PTS) Is the system time-invariant?
(c) (5 PTS) Is the system causal?
(d) (10 PTS) Is the system BIBO stable?
(e) (5 PTS) How would your answers to (a)-(d) change if the interval \( n \geq 0 \) were replaced by \( n \leq 0 \)?

2. A causal system is described by the difference equation

\[ y(n) - \frac{1}{2} y(n-1) = x^2(n), \quad n \geq 0 \]

with initial condition \( y(-1) = 2 \), and where \( x(n) \) denotes the input sequence.

(a) (5 PTS) Draw a block diagram representation of the system.
(b) (5 PTS) Find the zero-input solution of the system.
(c) (10 PTS) Find the zero-state solution of the system corresponding to \( x(n) = (1/2)^n u(n-1) \).
(d) (10 PTS) Find the complete solution of the system. Verify that your solution satisfies the initial condition and the difference equation.
(e) (15 PTS) Find the z–transform of the sequence \( nx(-n) + x^2(n-2) \). Specify its region of convergence. Find also the energy of this sequence.
(f) (5 PTS) Plot the sequence \( nx(-n) + x^2(n-2) \).

3. (10 PTS) Use the z–transform to evaluate the series

\[ \sum_{n=2}^{\infty} n^2 \left( \frac{1}{2} \right)^n u(n-1) \]