FINAL EXAMINATION
(Open Book)

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1. (20 PTS) A causal system is described by the second-order difference equation

\[ y(n) - \frac{7}{6} y(n - 1) + \frac{1}{3} y(n - 2) = x(n) - \frac{1}{2} x(n - 1), \quad y(-1) = 1, \ y(-2) = 0 \]

Find its complete response to \( x(n) = \left( \frac{1}{2} \right)^n u(n) \):

(a) (10 PTS) Using the unilateral \( z \)-transform.

(b) (10 PTS) Using the bilateral \( z \)-transform.
2. (20 PTS) A relaxed, causal, and stable system is described by the first-order difference equation

\[ y(n) - ay(n-1) = x(n) \]

where \( x(n) \) denotes the input sequence and \( y(n) \) denotes the output sequence.

(a) (10 PTS) Find all values of the scalar \( a \) in order to guarantee that a unit-amplitude tone at 750Hz that is sampled at twice its Nyquist rate is attenuated by \( 2/\sqrt{7} \).

(b) (5 PTS) Find all values of the scalar \( a \) so that the energy of the impulse response sequence of the system is equal to \( 4/3 \).

(c) (5 PTS) Find the value of the scalar \( a \) so that the response of the system to \( x(n) = u(n) \) is

\[ y(n) = \left[ 2 - \left( \frac{1}{2} \right)^n \right] u(n) \]

Justify all your answers completely.
3. (30 PTS) Consider the block diagram shown in the figure below where the LTI system is a lowpass filter. The DTFTs of the sequences at the points A, C, and E are also shown.

(a) (10 PTS) Find the frequency response and the impulse response of the unknown LTI system. Is the LTI system causal?

(b) (5 PTS) Plot the DTFT of the sequences at points B and D.

(c) (5 PTS) Find $\omega_o$.

(d) (5 PTS) Evaluate $x(n) * y(n)$.

(e) (5 PTS) Find the energy of the output sequence $y(n)$. 

\begin{align*}
\text{LTI system} & \quad \text{A} \\
\circ \quad \cos\left(\frac{\pi}{4}n\right) & \quad \omega \\
\circ \quad (-1)^n & \quad \omega_o n \\
\end{align*}

\begin{align*}
X(e^{j\omega}) & \quad X(e^{j\omega}) \\
& \quad \omega \\
C & \quad \omega \\
E & \quad \omega \\

(\text{a}) \quad \text{(10 PTS) Find the frequency response and the impulse response of the unknown LTI system. Is the LTI system causal?} \\
(\text{b}) \quad \text{(5 PTS) Plot the DTFT of the sequences at points B and D.} \\
(\text{c}) \quad \text{(5 PTS) Find } \omega_o. \\
(\text{d}) \quad \text{(5 PTS) Evaluate } x(n) * y(n). \\
(\text{e}) \quad \text{(5 PTS) Find the energy of the output sequence } y(n).
4. (30 PTS) Let \( x(n) \) be the \( N \)-point sequence \( \{x(0), x(1), \ldots, x(N-1)\} \) and let \( X(k) \) denote its \( N \)-point DFT sequence. Define the sequence \( x_1(n) \) of length \( 2N \) that is constructed from \( x(n) \) as follows:

\[
x_1(n) = \{x(0), 0, x(1), 0, \ldots, 0, x(N-1), 0\}
\]

In other words, a zero is added following each sample of \( x(n) \). This operation is known as interpolation and it is usually indicated in block diagram form as follows:

```
\[
x(n) \quad \uparrow 2 \quad x_1(n)
\]
```

Define also the extended DFT sequence

\[
X_2(k) = \{X(0), \ldots, X(N-1), 0, \ldots, 0\}
\]

That is, \( N \) zeros are appended to \( X(k) \).

(a) (10 PTS) Find the \( 2N \)-point DFT of \( x_1(n) \) in terms of the samples of \( X(k) \).

(b) (10 PTS) Find the even samples of the inverse \( 2N \)-point DFT of \( X_2(k) \) in terms of \( x(n) \).

(c) (10 PTS) Let \( x(n) \) and \( y(n) \) be \( N \)-point sequences with \( N \)-point DFTs \( X(k) \) and \( Y(k) \), respectively. Let \( z(n) \) be the output of the block diagram shown below. Express \( z(n) \) in terms of \( x(n) \) and \( y(n) \).