Appendix A
Exercise Problems

EXERCISE 1.1

If the surface concentration is fixed at the solid solubility limit of boron in silicon of $N_s = 1.8 \times 10^{20}$ atoms/cm$^3$ and boron is diffused at 950 °C for 30 minutes ($D_{boron} = 3.0 \times 10^{-15}$ cm$^2$/s at that temperature) into an n-type substrate, calculate the boron impurities at 0.05 μm and 0.1 μm.

Solution:
Since 30 minutes = 30 × 60 = 1800 s, and with a constant surface concentration, the distribution is a complementory error function,

\[
N(0.05 \mu, 1800) = 1.8 \times 10^{20} \text{erf}c\left[\frac{0.05 \times 10^{-4}}{2\sqrt{3} \times 10^{-15} \times 1800}\right]
\]

\[
N(0.05 \mu, 1800) = 1.8 \times 10^{20} \text{erf}c[1.0758] \quad \text{where erf}c[1.0758] = 0.12816
\]

from Table 1.1 or Figure Pl.1, therefore

\[
N(0.05 \mu, 1800) = 1.8 \times 10^{20} \text{erf}c[1.0758] = 2.3068 \times 10^{10}/\text{cm}^3
\]

Similarly,

\[
N(0.1 \mu, 1800) = 1.8 \times 10^{20} \text{erf}c[2.152] = 4.2169 \times 10^{17}/\text{cm}^3
\]

Note that the impurities have decreased by two orders of magnitude while the distance has only doubled.

EXERCISE 2.1

For the energy band diagram shown:
(a) Sketch the charge density, electric field, and potential.
(b) Let $kT = 0.026$ eV and calculate the maximum electric field and the built-in potential, $V_b$. 

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Solution:

\[ E_a \]

\[ 30 \, kT \]

\[ E_i \]

\[ 10 \, kT \]

\[ E_F \]

\[ 10 \, kT \]

\[ E_s \]

\[ 30 \, kT \]

Charge density

\[ 10^{-4} \, \text{cm} \]

Electric field

\[ \frac{dE_x}{dx} = \frac{1}{q} \left[ \frac{20 \, kT}{10^{-4}} \right] = -5.2 \, \text{KV/cm} \]

\[ V(x) \]

\[ 20 \, kT/q = 0.52 \, \text{volts} \]

**EXERCISE 2.2**

Sketch the energy band diagram (in units of \( kT \)), charge density, and potential for a silicon \( n^+ - p \) junction with \( N_{D} = 2 \times 10^{17} / \text{cm}^3 \) and \( N_A = 5 \times 10^{15} / \text{cm}^3 \). Let \( kT = 0.026 \, \text{eV} \) or \( kT/q = 0.026 \, \text{volts} \) and \( n_i = 10^{10} / \text{cm}^3 \).

Solution:

\[ V_{bi} = 0.026 \ln \left( \frac{10^{17}}{10^{15}} \right) = 29.934 \, kT = 0.77827 \, \text{volts} \]

\[ E_F - E_i = kT \ln \left( \frac{2 \times 10^{17}}{10^{15}} \right) = 16.811 \, kT \]

\[ E_E - E_i = -kT \ln \left( \frac{5 \times 10^{15}}{10^{16}} \right) = -13.122 \, kT \quad \text{and} \quad E_a = 43.08 \, kT \]
EXERCISE 2.3

For the example at the end of Section 2.3, calculate the junction voltage required to make $W$ equal to

(a) $2 \, \mu m$

Solution:

$$2 \times 10^{-4} = \left[ 1.306 \times 10^7(0.659 - V_A) \frac{1.1 \times 10^{16}}{10^{11}} \right]^{1/2}$$

$$0.659 - V_A = 2.7844 \quad \text{and therefore } V_A = -2.1254 \, \text{volts}$$

(b) $0.6 \, \mu m$

Solution:

$$0.659 - V_A = \frac{0.36 \times 10^{-8}}{1.306 \times 10^7(1.1 \times 10^{-15})} = 0.25059 \quad \text{and } V_A = 0.40841$$
EXERCISE 3.1

A silicon step junction has \( N_A = 10^{18}/\text{cm}^3 \) and \( N_D = 10^{15}/\text{cm}^3 \) with \( n_i = 10^{10}/\text{cm}^3 \) and \( kT = 0.026 \text{ eV} \). Calculate

(a) \( \Delta p_d(x_d) \) if \( V_A = 0.4 \) and 0.6 volts

Solution:

\[ n_{p0} = \frac{n_i^2}{N_A} = 10^{20} \text{ cm}^{-3} \]
\[ p_{p0} = \frac{n_i^2}{N_D} = 10^{20} \text{ cm}^{-3} \]
\[ \Delta p_d(x_d) = 10^5(e^{0.4\times0.026} - 1) = 4.581 \times 10^{11} \text{ cm}^{-3} \]
\[ \Delta p_d(x_d) = 10^5(e^{0.6\times0.026} - 1) = 1.052 \times 10^{15} \text{ cm}^{-3} \]

(b) \( \Delta n_d(-x_d) \) if \( V_A = 0.4 \) and 0.6 volts

Solution:

\[ \Delta n_d(-x_d) = 10^5(e^{0.4\times0.026} - 1) = 4.581 \times 10^{10} \text{ cm}^{-3} \]
\[ \Delta n_d(-x_d) = 10^5(e^{0.6\times0.026} - 1) = 1.052 \times 10^{11} \text{ cm}^{-3} \]

(c) Are low-level injection conditions valid in parts (a) and (b)?

In part (a) at \( V_A = 0.6 \) it is not valid since \( \Delta p_d(x_d) \) is not \( \ll 10^9 \text{ cm}^{-3} \). All the other cases are valid solutions.

EXERCISE 3.2

A step junction has \( N_A = 10^{17}/\text{cm}^3 \) and \( N_D = 5 \times 10^{15}/\text{cm}^3 \) with \( D_n = 30 \text{ cm}^2/\text{s} \), and \( D_p = 12 \text{ cm}^2/\text{s} \). Let \( n_i = 10^{10}/\text{cm}^3 \), \( kT = 0.026 \text{ eV} \), \( L_N = 10 \times 10^{-4} \text{ cm} \), \( L_P = 15 \times 10^{-4} \text{ cm} \), and \( A = 10^{-4} \text{ cm}^2 \). Calculate:

(a) the ratio of the hole current to the total current in the depletion region.

Solution:

\[ I_P(x_d) = qAN_i^2 \left[ \frac{D_P}{L_P N_D} \right] \left( e^{\frac{qV}{kT}} - 1 \right) \]
\[ I(x_d) = qAN_i^2 \left[ \frac{D_P}{L_P N_D} + \frac{D_N}{L_N N_A} \right] \left( e^{\frac{qV}{kT}} - 1 \right) \]
\[ \frac{I_P}{I} = \frac{1}{1 + \frac{D_N L_P N_D}{D_P L_N N_A}} = \frac{1}{1 + \frac{30 \times 1.5 \times 10^3}{12 \times 10}} = \frac{1}{1 + 0.1875} = 0.8421 \]
or 84.2% of the total.

(b) Repeat part (a) for the electron current.

Solution:

\[ I_e = \frac{1}{1 + \frac{D_n L_n N_A}{D_p L_p N_p}} = \frac{1}{1 + \frac{12 \times 10}{30 \times 15} \times 10^{17}} = 0.13789 \]

or 15.8% of the total.

(c) If \( N_D \) were made smaller, discuss how this would affect the ratios of parts (a) and (b).

As \( N_D \) decreases the hole current percentage will increase and the electron current will decrease their percentage of the total.

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**EXERCISE 3.3**

For the data given in Exercise 3.2 calculate the following at \( V_A = 0.4 \):

(a) \( \Delta p(x') = 0 \) and \( \Delta p(x') = 60 \, \mu \text{m} \)

\[ \Delta p(x') = p_0 (e^{x' / \lambda_T} - 1)e^{-x' / \lambda_T} = \frac{n_i^2}{N_D} (e^{x' / 0.025} - 1)e^{-x' / 0.025} = 9.6047 \times 10^{10} e^{-0.025(15 \times 10^{-10})} \]

\[ \Delta p_n(0') = 9.6047 \times 10^{10} / \text{cm}^3 \text{ and } \Delta p_e(x' = 60 \times 10^{-4}) = 1.759 \times 10^{9} / \text{cm}^3 \]

(b) \( I_p(x' = 0) \) and \( I_p(x' = L_p) \)

\[ I_p(x') = q A n_i \left[ \frac{D_p}{L_p} \frac{1}{N_D} \right] (e^{x' / \lambda_T} - 1)e^{-x' / \lambda_T} = 1.024 \times 10^{-10} (e^{15.385} - 1)e^{-15.385(15 \times 10^{-10})} \]

\[ I_p(x' = 0) = 4.9195 \times 10^{-11} \text{ and } I_p(x' = L_p) = 4.9195 \times 10^{-11} e^{-1} = 1.8098 \times 10^{-10} \]

(c) If the recombination rate were, for some reason, to quadruple in the n-region, calculate the new value of \( I_p(x' = 0) \).

\( \tau_n \) becomes \( \tau_n / 4 \) and hence \( L_p \) becomes \( L_p / 2 \), and since \( I_p(x') \) is proportional to \( 1/L_p \) it will double. Therefore \( I_p(x' = 0) \) will double to 9.839 \( \times 10^{-10} \).

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**EXERCISE 4.1**

For a silicon p-n step junction doped at \( N_A = 10^{17} / \text{cm}^3 \) and \( N_D = 5 \times 10^{15} / \text{cm}^3 \) with \( n_i = 10^{10} / \text{cm}^3 \), \( kT/q = 0.026 \) volts, \( m = 4.0 \), and \( \xi_c = 4 \times 10^5 \) V/cm:

(a) Calculate \( V_{sat} \).
\[ V_{BR} = (4 \times 10^5)^2 \left( \frac{8.854 \times 10^{-14} \times 11.8}{2q} \right) \left( \frac{10^{15} + 5 \times 10^{15}}{10^{17} \times 5 \times 10^{15}} \right) = 109.7 \text{ volts} \]

(b) At what value of \( V_A \) will the current have increased by

(i) 10.

(ii) 100?

\[ M = \frac{10}{1 - \left( \frac{V_A}{V_{BR}} \right)^4} \quad \text{or} \quad \left( \frac{V_A}{V_{BR}} \right)^4 = \frac{M - 1}{M} \]

\[ V_A = V_{BR} \sqrt[4]{\frac{M - 1}{M}} = 109.7 \sqrt[4]{\frac{10 - 1}{10}} = 106.85 \]

\[ V_A = 109.7 \sqrt[4]{\frac{100 - 1}{100}} = 109.42 \]

(c) What is the ratio of \( I_{R-c}(V_A = -20)/I_{R-c}(V_A = -2)\)?

\[ V_{Bi} = 0.7603 \quad \text{and} \quad W = k(V_{Bi} - V_A)^{1/2} \]

\[ I_{R-c}(-20) = \frac{0.7603 + 20}{0.7603 + 2} = 2.7424 \]

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**EXERCISE 4.2**

For a \( p^-n \) step junction that has \( W = 10^{-4} \text{ cm} \), \( n_i = 10^{10}/\text{cm}^3 \), \( N_0 = 5 \times 10^{15}/\text{cm}^3 \), \( A = 10^{-4}/\text{cm}^2 \), \( kT = 0.026 \text{ eV} \), \( \tau_0 = 20 \mu\text{s} \), and \( D_r/L_r = 10^2 \) with \( n_1 = 1.05 \) and \( n_2 = 2.0 \), what is:

(a) The ratio of \( I_{Rec}/I_{diffusion} \)

\[ I_{diffusion} = qA n_i^2 \left( \frac{D_r}{L_r N_0} \right) \left( e^{\frac{V_A}{n_i/kT}} - 1 \right) \]

\[ I_{Rec} = qA n_i W \left( e^{\frac{V_A}{n_i/kT}} - 1 \right) \]

\[ \frac{I_{Rec}}{I_{diffusion}} = \frac{4 \times 10^{-11}(e^{0.023/kT} - 1)}{3.2 \times 10^{-15}(e^{0.23/kT} - 1)} = \frac{125(e^{0.23/V_A} - 1)}{10} \]

(b) The value of the ratio at \( V_A = 0.05 \), \( V_A = 0.1 \), and \( V_A = 0.2 \): From above,

\[ \frac{I_{Rec}}{I_{diffusion}} = \frac{125(e^{0.23/kT} - 1)}{0.23/kT} = 38.519 \]
\[
\frac{I_{\text{Rec}}}{I_{\text{diffusion}}} = \frac{125(e^{0.211\times0.1} - 1)}{(e^{0.63\times0.1} - 1)} = 19.228
\]
\[
\frac{I_{\text{Rec}}}{I_{\text{diffusion}}} = \frac{125(e^{0.211\times0.2} - 1)}{(e^{0.63\times0.2} - 1)} = 3.7717
\]

**EXERCISE 5.1**

A p−n step junction is doped \(N_A = 10^{15}/\text{cm}^3\) and \(N_D = 10^{13}/\text{cm}^3\). Let \(kT = 0.026\) eV, \(n_i = 10^{10}/\text{cm}^3\), \(A = 10^{-4}\) cm², \(L_N = 14 \times 10^{-4}\) cm, \(L_P = 35 \times 10^{-4}\) cm, \(D_N = 20\) cm²/s, and \(D_P = 12.5\) cm²/s.

(a) Calculate the depletion capacitance if \(V_A = 0\) and \(-2\) volts.

\[V_n = 0.6585 \quad \text{and} \quad W = \sqrt{1.306 \times 10^7(0.6585) \left[\frac{1.1 \times 10^{16}}{10^3}\right]} = 0.9727 \times 10^{-4}\text{ cm}\]

\[C_P = \frac{K_0 e_A}{W} = \frac{11.8 \times 8.854 \times 10^{-11} \times 10^{-4}}{0.9727 \times 10^{-4}} = 1.074 \text{ pF}\]

\[C_J(V_A = -2) = \frac{1.074 \text{ pF}}{\left[1 - \left(-\frac{2}{0.6585}\right)\right]^{1/2}} = 0.5345 \text{ pF}\]

(b) Calculate the low-frequency conductance at \(V_A = 0\) and \(-2\) volts.

\[G_0 = \frac{q(T + I_0)}{kT} = \frac{q}{kT} \left\{ qAN_i \left[\frac{D_P}{L_P N_0} + \frac{D_N}{L_N N_A} \right] e^{qV_A/kT} \right\}\]

\[G_0 = \frac{8 \times 10^{-15}}{0.026} e^{0.026}\]

\[G_0 = 3.077 \times 10^{-11} \quad \text{at} \quad V_A = 0\]

\[G_0 = 3.077 \times 10^{-11} e^{-0.026} = 1.2046 \times 10^{-10}\]

**EXERCISE 5.2**

If for a p-n junction \(jv\tau_p = 1\) and \(jv\tau_n = 0.90\) at \(\omega = 10^5\) rad/s with \(A = 10^{-4}\) cm², \(V_A = 0.6\) volts, and \(\tau_p = 1.6 \times 10^{-10}\)

\[\frac{qD_n}{L_N} = 5.0 \times 10^{-11}\]

\[\frac{qD_p}{L_P} \eta_n = 1.6 \times 10^{-10}\]
Calculate $G$ and $C_D$.

$Y = \frac{g_A}{kT} \left[ 1.6 \times 10^{-10}\sqrt{1 + ji} + 5.0 \times 10^{-11}\sqrt{1 + j0.90} \right] e^{0.60.936}$

$Y = 8.647 \times 10^{4}[1.6 \times 10^{-10}(1.414 \angle 45^\circ)^{1.2} + 5 \times 10^{-11}(1.345 \angle 41.99^\circ)^{1.2}]$

$Y = 8.647 \times 10^{4}[2.995 \times 10^{-10} + j0.9359 \times 10^{-10}]$

$Y = 1.9884 \times 10^{-4} + j0.80927 \times 10^{-4}$

$G = 1.9884 \times 10^{-4}$ and $C_D = \frac{0.80927 \times 10^{-4}}{10^4} = 0.80927$ pF

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**EXERCISE 6.1**

The $p^+ - n$ junction has $I_f = 1$ mA and $\tau_p = 1$ $\mu$s and is turned off by $I_R = 0$. Derive an equation for $Q_p(t)$.

\[
\frac{dQ_p(t)}{dt} = 0 - \frac{Q_p(t)}{\tau_p}
\]

\[
\frac{dQ_p(t)}{Q_p(t)} = \frac{dt}{\tau_p}
\]

\[
\ln[Q_p(t)]_{Q_p(0)}^{Q_p(t)} = -\left[ \frac{t}{\tau_p} \right]^{1}
\]

\[
Q_p(t) = Q_p(0)e^{-t/\tau_p}
\]

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**EXERCISE 6.2**

For the turn-on transient by a constant current, derive a formula for the time required to reach 90% of the total current.

At infinite time, $v_A = V_A$ and 90% is 0.9 $kT/q \ln[I_f/I_0]$. In Eq. (6.21b) the "-1" term is very small compared to the exponent term as $t$ approaches infinity. Then Eq. (6.22) becomes

\[
0.9V_A = \frac{kT}{q} \ln \left[ \frac{I_f}{I_0}(1 - e^{-\psi_T}) \right] = 0.9 \left[ \frac{kT}{q} \ln \left( \frac{I_f}{I_0} \right) \right]
\]

\[
\frac{I_f}{I_0}(1 - e^{-\psi_T}) = \left( \frac{I_f}{I_0} \right)^{0.9}
\]

\[
\psi_T = \frac{t}{\tau_p} \ln \left[ 1 - \left( \frac{I_f}{I_0} \right)^{0.1} \right]
\]
EXERCISE 7.1

A metal with $\Phi_M = 4.75$ eV (Au) and a semiconductor (Si) with $\chi = 4.05$ eV are formed into an ideal metal–semiconductor contact. If $kT = 0.026$ eV, $n_i = 10^{10}/\text{cm}^3$ and $N_D = 10^{16}/\text{cm}^3$.

(a) Is it a Schottky barrier or an “ohmic” contact?

If $\Phi_M > \chi + (E_c - E_{\text{Fermi}})$ then it is a Schottky diode

$$ (E_{\text{Fermi}} - E_F) = kT \ln \left[ \frac{N_D}{n_i} \right] = (0.26) \ln \left[ \frac{10^{16}}{10^{10}} \right] = 0.3592 \text{ eV} $$

$$ (E_c - E_{\text{Fermi}})_{\text{bulk}} = 0.56 - 0.3592 = 0.2008 \text{ eV} $$

Hence, $\Phi_S = 4.05 + 0.2008 = 4.2508$ eV which is less than $4.75$ eV = $\Phi_M$.

(b) Calculate the ideal barrier height for an electron in the metal at $E_{\text{Fermi}}$.

$$ \Phi_M - \chi = 4.75 - 4.05 = 0.700 \text{ eV} = \Phi_B $$

(c) Calculate the ideal barrier height for an electron at $E_c$ in the semiconductor.

$$ \eta V_n = \Phi_M - \chi - (E_c - E_{\text{Fermi}})_{\text{bulk}} $$

$$ = 4.75 - 4.05 - 0.2008 = 0.4992 \text{ eV} $$

$$ V_n = 0.4992 \text{ volts} $$

(d) Calculate $x_n$ for $V_A = 0$ and $V_A = -2$ volts

$$ x_n = \sqrt{\frac{2kT_0(V_n - V_A)}{\eta N_D}} $$

$$ x_n = \sqrt{\frac{2(1.18)8.85 \times 10^{-14}}{1.6 \times 10^{-19} \times 10^{16} (0.4992 - V_A)}} $$

$$ x_n = 36.116 \times 10^{-6} \sqrt{0.4992 - V_A} \quad \text{at } V_A = 0 \quad x_n = 25.57 \times 10^{-6} $$

$$ x_n = 0.255 \times 10^{-4} \quad x_n = 0.255 \mu\text{m} $$

@ $V_A = -2$

$$ x_n = 57.095 \times 10^{-6} = 0.57095 \times 10^{-4} = 0.57095 \mu\text{m} $$

EXERCISE 7.2

A Schottky barrier diode has a value of $\Phi_B = 0.5$ eV, $I_S = 5 \times 10^{-12}$ A, $n = 1.07$, and $kT = 0.026$ eV. If a second device were to be made with $\Phi_B = 0.7$ eV with everything else the same, what is the current through both if $V_A = 0.4$ volts and $V_A = -2$ volts?
\[ I_1 = 5 \times 10^{-12}(e^{0.410.0260.026} - 1) = 5 \times 10^{-12}(e^{35.945-V_A} - 1) \]

\[ I_1 = 8.7765 \mu A \quad @ \quad V_A = 0.4 \quad I_1 = x \times 10^{-12} \mu A \quad @ \quad V_A = -2 \]

since

\[ I_{11} = K Ae^{-0.026} = KAe^{-0.520.026} = 5 \times 10^{-12} \quad \text{then} \quad KA = 1.1241 \times 10^{-3} \]

\[ I_{22} = 1.1241 \times 10^{-7}e^{-0.790.026} = 2.2816 \times 10^{-15} \mu A \]

\[ I_2 = 2.2816 \times 10^{-15}(e^{35.945-V_A} - 1) \quad @ \quad V_A = 0.4 \text{ volts} \]

\[ I_2 = 2.2816 \times 10^{-15} \quad @ \quad V_A = -2 \text{ volts} \]