For your linearized mathematical model in the form:

\[ \frac{dx(t)}{dt} = Ax(t) + Bu(t), \]
\[ y(t) = Cx(t) + Du(t); \]

or

\[ x(k+1) = Hx(k) + Gu(k), \]
\[ y(k) = Cx(k) + Du(k), \]

(a) determine the eigenvalues and their corresponding eigenvectors of A or H. Is A or H simple?

(b) rewrite your model equations in the canonical form, and determine whether there are modes that are unaffected or weakly affected by the input u or do not appear in the output y. Are there any unstable modes that are unaffected by u or do not appear in y?

(c) investigate the effect of parameter variations on the answer to part (b) (perturb one or more elements of A or H matrix). (See Section 3.3 of Lecture Notes 9) The choice of the perturbed parameters should be based on physical considerations, i.e. Which parameter values are not precisely known.

(d) From your result in part (c) on the canonical form, decompose the systems into subsystems \( S_{cc}, S_{qf}, S_{qq}, \) and \( S_{qq}. \) Identify any unstable modes which are uncontrollable and/or unobservable. If there is a unstable mode which is controllable, find a linear state feedback control to stabilize it.

(You may use MATLAB or other software to calculate the eigenvalues and eigenvectors)

(TURN IN THE MINI-PROJECT REPORT ON OR BEFORE MARCH 24, Thursday)