Back body factor

What happens if we have non-zero $V_{ssub}$?

Now $V_{th}$ is always defined as the $V_{gs}$ at the on-set of strong inversion in the channel for small $V_{ds}$. If $V_{ssub} = 0$, near the source the band diagram looks like:

\[ V_{ssub} < V_{s} \]

That is \[ V_{s} = 2|\Phi_{b}| + |V_{ssub}| \] at onset of strong inversion.

\[ V_{ssub} = V_{s} + V_{FB} + \frac{C_{ox}}{C_{ox}} \]

with \[ C_{ox} = |2|e|/8NA V_{s} \]
at onset of strong inversion

\[ V_{\text{onsub}} = 2\langle \Phi \rangle + V_{\text{ssub}} + V_{\text{FB}} + \sqrt{2E_i \cdot g_N A(2\langle \Phi \rangle + V_{\text{ssub}})} \]

\[ \frac{1}{\text{Cox}} \]

\[ V_{\text{th}} = V_{\text{Fs}} \text{ at strong inversion} \]

\[ V_{\text{th}} = V_{\text{ssub}} - V_{\text{ssub}} \]

\[ V_{\text{th}} = V_{\text{FB}} + 2\langle \Phi \rangle + \sqrt{2E_i \cdot g_N A(2\langle \Phi \rangle + V_{\text{ssub}})} \]

\[ \frac{1}{\text{Cox}} \]

\[ V_{\text{th}0} + \sqrt{2E_i \cdot g_N A} \left( \sqrt{2\langle \Phi \rangle + V_{\text{ssub}}} - \sqrt{2\langle \Phi \rangle} \right) \]

where \( V_{\text{th}0} \) is \( V_{\text{th}} \) with \( V_{\text{ssub}} = 0 \)

Now \( \frac{2V_{\text{th}}}{2\langle \Phi \rangle + V_{\text{ssub}}} \), with \( V_S = 0 \)

is given by \( \frac{\sqrt{2E_i \cdot g_N A}}{2 \text{ Cox} \sqrt{2\langle \Phi \rangle + V_{\text{ssub}}}} = \sqrt{\frac{E_i \cdot g_N A}{2 \text{ Cox} (2\langle \Phi \rangle + V_{\text{ssub}})}} \)
The higher the substrate doping, the more sensitive is \( V_{th} \) to \( V_{sub} \).

**Subthreshold current.**

Now we have:

\[
I_D = \frac{W}{L} C_{ox} \mu_n (V_0 - V_{th} - \frac{V_D}{2}) V_D \quad \text{for small } V_0 \text{ and } E_0 > V_{th}
\]

\[
= \frac{W}{2L} C_{ox} \mu_n (V_0 - V_{th})^2 \quad \text{for large } V_0 \text{ and } V_0 > V_{th}
\]

and \( V_{osat} \) (the \( V_0 \) at which we change) from (1) to (2) is given by \( V_D = V_0 - V_{th} \).

This means what if \( V_0 < V_{th} \)? Is \( I_D = 0? \)

No, there is not a discontinuity in the transistor \( I-V \) characteristics.
Now if \( V_0 < V_{th} \) and assume \( V_{sat} = 0 \)
\[ N_s(0) < 2 \Phi_B \]

\[ N_s(0) = \frac{8 \Phi(0) - 2 \Phi_B}{kT} \]

i.e. \( N_s(0) = N_A e^{\frac{8 \Phi(0) - 2 \Phi_B}{kT}} \)

\[ N_s(L) = N_A e^{\frac{8 \Phi(L) - 2 \Phi_B - V_D}{kT}} \]

i.e. \( N_s(L) = N_A e^{\frac{8 \Phi(L) - 2 \Phi_B - V_D}{kT}} \)

What about \( Q_i(0) \)?

\[ Q_i(0) = 8 \int_0^L N(0) dy \]

Now near the Si/SiO₂ interface the band looks like

slope is \( \eta \)

The E-field at the interface always has a straight line from 0 to \( L \) as long as \( t \) is small.

(Taylor's Theorem)
Since $Q_1(\xi)$ is small (at all $x$)

- electron current = drift + diffusion
  \[ \propto \text{diffusion} \]
  \[ \text{as drift} \propto \Delta \xi \]

i.e., by Kirchhoff's law $I = f(x)$

\[ I = WD \frac{Q_1(L) - Q_1(0)}{L} \]

Since current is small $\Psi_1(x)$ is very flat until close to $L$.

\[ I \approx -\frac{\alpha_{np} N_0 V_F}{\Delta} e^{\frac{E_{V_F}(0) - 2kT}{kT}} \left( 1 - e^{\frac{-8V_D}{kT}} \right) \]
But we really want $V_6$, NOT $V_S$.

Now let us expand around $V_S = 1.5/I_{B1}$.

\[ \Phi V_{6*} = V_6 \left( V_S = 1.5/I_{B1} \right) = 1.5/I_{B1} + V_{FB} + \frac{\sqrt{2C_{N1}V_{N1}}}{C_{ox}} \]

\[ V_6 = V_{6*} + 2V_S(V_6 - V_{6*}) \]

\[ V_S = 1.5/I_{B1} + \frac{2V_S}{2V_6} \left( V_6 - V_{6*} \right) \]

But \( \frac{2V_S}{2V_6} \) is just a capacitor divider.

i.e. \( \frac{2V_S}{2V_6} = \frac{C_{ox}}{C_{ox} + C_S^*} \)

with $C_S^*$ evaluated around $V_S = 1.5/I_{B1}$.

Let \( \frac{C_{ox} + C_S^*}{C_{ox}} = N \).
Put this together, we have

\[ I_D \approx -8 \frac{W}{L} D_n\left(\frac{V_T}{V}\right) N_A e^{-\frac{8 V_T}{kT}} \times \left(1 - e^{-\frac{8 V_D}{kT}}\right) \]

\[ = -8 \frac{W}{L} D_n\left(\frac{V_T}{V}\right) N_A e^{-\frac{8V_T}{2kT}} e^{\frac{8(V_s)}{kT}} e^{-\frac{8V_s}{kT}} \left(1 - e^{-\frac{8 V_D}{kT}}\right) \]

\[ = -8 \frac{W}{L} D_n\left(\frac{V_T}{V}\right) N_A e^{-\frac{8V_T}{2kT}} e^{\frac{8(V_s)}{kT}} \left(1 - e^{-\frac{8 V_D}{kT}}\right) \]

\[ = I^b \left(1 - e^{-\frac{8 V_D}{kT}}\right) \]

\[ I_D \propto e^{\frac{8 V_s}{kT}} \quad \text{and} \quad I_D \text{ is independent of } V_0 \quad \text{for } V_0 \geq \frac{8}{g} \]

But \( \eta = \frac{C_0 + C_s}{C_0} \quad \Rightarrow \quad \eta > 1 \)

\[ I_D \text{ versus } V_0 \text{ is less steep compared to} \]