Q.1

(a) 2nd collector of Q1 connect to Vcc

Based on KCL:

\[ \text{In} + \frac{g_m}{R_i} \text{Vin} + \frac{V_n}{R_i} = \frac{V_n}{R_i} \]

\[ \text{In} + g_m (-V_n) + \frac{-V_n}{R_i} = \frac{V_n}{R_i} \]

\[ \text{In} = \left( g_m + \frac{1}{R_i} + \frac{1}{R_i} \right) V_n \]

So \( R_i = \frac{V_n}{\text{In}} = \frac{1}{g_m + \frac{1}{R_i} + \frac{1}{R_i}} = R_i \parallel \left( \frac{1}{g_m + \frac{1}{R_i}} \right) \)

(b) Small signal equivalent

Based on KCL at the super node 1:

\[ \frac{V_n}{R_{n1}} + g_{m1} V_1 = \frac{V_2}{R_{n2}} + g_{m2} V_2 \quad \cdots \quad 1 \]

Since \( V_n = V_1 + V_2 \)

\[ \text{In} = \frac{V_1}{R_{n1}} \quad \cdots \quad 2 \]

Solve 1, 2, 3 together, and

\[ R_i = \frac{V_n}{\text{In}} = \frac{V_{n1} + V_{n2} + (g_{m1} + g_{m2}) R_{n1} R_{n2}}{1 + g_{m2} R_{n2}} \]

\[ = R_{n1} + (1 + \beta) \left( \frac{1}{g_{m2} R_{n2}} \right) \]
Based on KCL at node 1:
\[ \frac{V_{n1}}{T_{n1}} + g_{m1} V_{in1} = \frac{V_{n2}}{T_{n2}} \quad \cdots \quad (1) \]

\[ Vin = V_{n1} + V_{n2} \quad \cdots \quad (2) \]

\[ Lin = \frac{V_{in1}}{T_{n1}} \quad \cdots \quad (3) \]

Solve (1), (2) and (3) together, getting rid of $V_{n1}$ and $V_{n2}$:

\[ Rin = \frac{V_{in}}{I_{in}} = \frac{V_{n1} + V_{n2}}{T_{n1}} \left(1 + g_{m1} T_{n1}\right) \]

In order to get output impedance, we apply a voltage source $V_i$ at the output, and measure the output current $I_x$. Carefully analyze this circuit and write equations based on KCL or KVL to find $\frac{V_i}{I_x} = R_{out}$. 

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**Small Signal Equivalent**
\[ V_1 + V_2 = 0 \]  
\[ V_X = V_3 + V_2 \]  
\[ \frac{V_1}{r_{m1}} + \frac{V_2}{r_{o1}} = V_3 \left( \frac{1}{r_{m2}} + \frac{1}{r_{o2}} \right) \]  
\[ I_X = \frac{V_2}{r_{o1}} + \frac{g_{m1} V_1}{r_{m2}} \]

Solve 1, 2, 3, 4 together, and getting rid of \( V_1, V_2, V_3 \).

We can get \( R_{out} = \frac{V_X}{I_X} = r_{o1} + \frac{1}{\frac{1}{r_{m1}} + \frac{1}{r_{m2}} + \frac{1}{r_{o2}}} \)

\[ = r_{o1} + \left( 1 + \frac{g_{m1}}{r_{o1}} \right) \left( \frac{1}{r_{m1}} + \frac{1}{r_{m2}} + \frac{1}{r_{o2}} \right) \]

KCL at node 1 \[ \frac{V}{r_{m1}} + \frac{g_{m1} V}{r_{o1}} + \frac{V_2}{r_{o2}} = \frac{V_3}{r_{o2}} \]  
\[ I_X = \frac{g_{m1} V}{r_{o1}} \]  
\[ V_X = V_3 + V_2 \]  
\[ V_1 + V_2 = 0 \]

Solve 1, 2, 3, 4 together,

\[ R_{out} = \frac{V_X}{I_X} = r_{o1} + \frac{1 + \frac{g_{m1}}{r_{o1}}}{\frac{1}{r_{m1}} + \frac{1}{r_{m2}}} \]

\[ = r_{o1} + \left( 1 + \frac{g_{m1}}{r_{o1}} \right) \left( \frac{1}{r_{m1}} + \frac{1}{r_{m2}} \right) \]
Assume $V$, $V_{BE}$ as shown on the figure.

Current $I_B = I_{B1} + I_{B2} = \frac{\beta_1}{\beta_2} I_{C1} + \frac{\beta_2}{\beta_1} I_{C2}$

$\beta_1 = \beta_2 = 100$

$I_{C1} = I_F \exp \frac{V_{BE}}{V_T}$

$I_{C2} = I_{S2} \exp \frac{V_{BE}}{V_T}$

So $I_B = \frac{1}{100} (I_{S1} + I_{S2}) \exp \frac{V_{BE}}{V_T}$

$kCL\ at\ node\ 1$

\[
\frac{V_{cc} - V_i}{R_1} = \frac{V_i}{R_2} + I_B
\]

So $\frac{2.5 - V_i}{13k} = \frac{V_i}{12k} + I_B$

$I_E = I_{E1} + I_{E2} = \frac{\beta_1}{\beta_2} I_{C1} + \frac{\beta_2}{\beta_1} I_{C2}$

$= (\frac{101}{100})(I_{S1} + I_{S2}) \exp \frac{V_{BE}}{V_T}$

So $V_i - V_{BE} = I_E \cdot R_E$

$V_i - V_{BE} = 404 \cdot \frac{101}{100} (I_{S1} + I_{S2}) \exp \frac{V_{BE}}{V_T}$

\[
\begin{align*}
V_i &= V_{BE} + 404 (I_{S1} + I_{S2}) \exp \frac{V_{BE}}{V_T} \\
V_{BE} &= V_i - 404 (\frac{2.5 - V_i}{130} - \frac{V_i}{120})
\end{align*}
\]

Use MATLAB to calculate $V_{BE} = 0.7263\,\text{V}$

So $I_{C1} = I_{S1} \exp \frac{V_{BE}}{V_T} = 5 \times 10^{-6} \exp \frac{0.7263}{0.025} = 0.68\,\text{mA}$

$I_{C2} = \frac{I_{C1}}{2} = 0.34\,\text{mA}$
 requirement is \( V_{BC} \leq 200\text{mV} \)

Since \( I_C = I_s \exp \frac{V_B}{V_T} \)

\[
I_B = \frac{1}{\beta} I_C = \frac{1}{100} I_s \exp \frac{V_B}{V_T}
\]

\[
V = V_B + I_B \cdot R_B
\]

\[
V_C = V_1 - R_P \cdot I_C = V_B + I_B R_B - R_P I_C \quad \cdots \quad (1)
\]

Since \( V_{BC} \leq 0.2 \) so \( V_B - V_C < 0.2 \text{V} \)

From (1), \( V_B - V_C = R_P I_C - I_B R_B < 0.2 \text{V} \)

So, \( R_B > \frac{R_P \frac{I_C}{I_B} - 0.2}{I_B} = R_P \beta - \frac{0.2}{I_B} \)

Since \( R_B \) is related \( I_B \), so we need to decide \( I_B \) first.

Minimum value of \( R_B = R_P \beta - \frac{0.2}{I_B} \) when \( I_B \) increase, minimum \( R_B \) also increase, so we should find them together.

From the circuit, we know

\[
I_E = I_C + I_B = \frac{V_{CE} - V_{BE} - R_B I_B}{1k}
\]

Since \( R_B I_B = R_P I_C - 0.2 \)

So

\[
1.5 I_C = \frac{2.7 - V_{BE}}{1k}
\]

\[
V_{BE} = V_T \ln \frac{I_C}{I_S} \quad \cdots \quad (2)
\]

Assume initial \( V_{BE} = 0.6 \text{V} \), we do the iteration for (1) (2).

Finally we get \( V_{BE} = 0.756 \text{V} \)

\[
I_B = 1.287 \times 10^{-4} \text{A}
\]

So \( R_B > R_P \beta - \frac{0.2}{I_B} = 34.4 \text{k}\Omega \)

When \( R_B \) increase, \( I_B \) will decrease, the minimum requirement \( R_B \) will decrease. So, beyond 34.4 \( \text{k}\Omega \) will be OK.
input impedance is $V_{in}$, because the input device is common emitter structure.

Output impedance:

Shorted input, small signal equivalent is shown.

Since $V_2 > 0$, $I_1 = \alpha_m V_1 = 0$

$V_x = I_x R_c - V_2$

$\begin{cases} I_x + \frac{V_2}{R_m} + \alpha_m V_2 = 0 \\
\end{cases}$

We solve

$R_{out} = \frac{V_x}{I_x} = R_c + \frac{1}{\alpha_m + \frac{1}{R_m}}$

$voltage\ gain:\n
As we know, for common Emitter structure, voltage gain $\text{Av} = \alpha_m R_c$, $R_c$ is the total impedance at collector. In our circuit it is the output impedance.

So $\text{Av} = \alpha_m \cdot R_{out}$

$= \frac{V_2}{\alpha_m R_m + 1}$

You can also use small signal analysis to get the same result.
Input impedance is same as part (a), which is $R_{\text{in}}$.

Output impedance:

Equivalent small signal is shown as following:

Similar analysis method as part (a)

$$V_x = I_x R_C + V_i$$

$$\frac{V_i}{R_{\text{in}}} + g_m V_i = I_x$$

So,

$$R_{\text{out}} = \frac{V_x}{I_x} = R_C + \left( \frac{R_{\text{in}}}{g_m} \right)$$

$$= R_C + \frac{R_{\text{in}}}{1 + g_m R_{\text{in}}}$$

Voltage gain:

Same analysis as part (a)

So,

$$A_v = -g_m \cdot \left( R_C + \frac{R_{\text{in}}}{1 + g_m R_{\text{in}}} \right)$$
Again, for input impedance, the result is the same as part (a), (b), which is:
\[ R_{in} \]

Output impedance:
the small signal equivalent circuit is shown as:

\[ I_x = I_1 + I_2 = \frac{V_x}{R_{in}} + g_m V_x = \left( \frac{1}{R_{in}} + g_m \right) V_x \]

So, output impedance is:
\[ \frac{1}{R_{out}} = \frac{1}{1 + g_m R_{in}} \]

Voltage Gain:
The analysis for voltage gain is same as part (a), (b).

SO:
\[ Av = -g_m \cdot \frac{R_{out}}{1 + g_m R_{in}} \]
Output impedance:

\[ I_X = \frac{g_{m2}V_i}{R_c} + \frac{V_i}{R_{t2}} \]
\[ \frac{V_x - V_i}{R_c} = \frac{V_i}{R_{t2}} \]

So, \( R_{out} = \frac{V_x}{I_X} = \frac{R_c + \frac{1}{g_{m2}R_{t2}}}{1 + \frac{g_{m2}R_{t2}}{R_c}} \)

Voltage Gain:

Here, we can not directly use common emitter voltage gain equation as before, we need do small signal analysis.

At node 0:

\[ \frac{V_{out} - V_i}{R_c} = \frac{V_i}{R_{t2}} + g_{m1}V_i \] \( - \) 0

At output:

\[ g_{m2}V_i + \frac{V_{out} - V_i}{R_c} = 0 \] \( - \) 2

From equations 0 \& 2, we get:

\[ \frac{V_{out}}{V_i} = \frac{(g_{m2}R_c - 1)g_{m1}R_{t2}}{1 + g_{m2}R_{t2}} \]